Section 2.3: Tchebysheff's Theorem and the Empirical Rule

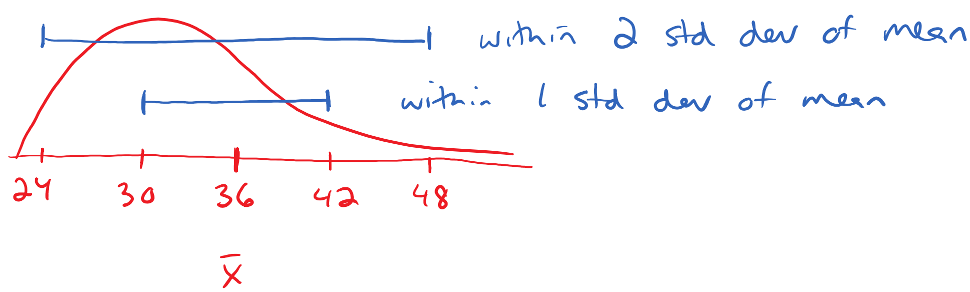
Friday, March 6, 2020 10:42 AM

or Chebyshev

Tchebysheff's Theorem: works for all distributions (symmetrical or skewed, unimodal or multimodal)

- for any set of measurements, <u>at least</u> $\left(1 - \frac{1}{k^2}\right)$ of the measurements will fall within k standard deviations of the mean for $k \geq 1$

example: consider a data set with sample mean $\bar{x} = 36$ and std dev $s = 6$.



What does Tchebysheff's theorem say about the number of measurements that lie between 24 and 48?

24 36 48

$\vdash\!\!\dashv$ 12 = 2 std dev

Tcheby says: for $k = 2$

$$1 - \frac{1}{k^2} = 1 - \frac{1}{4} = \frac{3}{4} = 75\%$$

at least 75% of measurements will lie
$\geq 75\%$

within the interval 24–48

so

k	$1 - 1/k^2$
1	0
1.5	5/9
2	3/4
2.5	21/25
3	8/9

ess

so $\geq 0\%$ * lie within $\bar{x} \pm 1\ s$

$\geq 55.\bar{5}\%$ $\bar{x} \pm 1.5\ s$

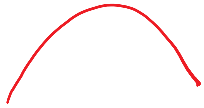
$\geq 75\%$ $\bar{x} \pm 2\ s$

$\geq 84\%$ $\bar{x} \pm 2.5\ s$

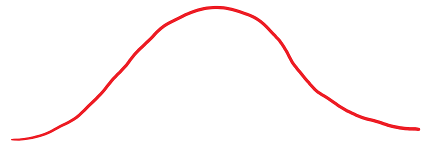
$\geq 88.\bar{8}\%$ $\bar{x} \pm 3\ s$

* this seems like a totally useless statement <u>but</u> what it is saying is that there could be <u>no</u> data with 1 std dev of mean

<u>Empirical Rule:</u> only works for "mound-shaped" or "bell-shaped" curves



mound



bell

unimodal

roughly symmetrical

ess

approximately 68% of measurements fall within $\bar{x} \pm 1\ s$

" 95% $\bar{x} \pm 2\ s$

" 99.7% $\bar{x} \pm 3\ s$