STAT 157 – Test 2: Version A

November 29, 2017 **Name:** Solution Set

Instructor: Patricia Wrean

Total: 30 points

1. (6 points) A computer system requires a case-sensitive, alphanumeric password containing 8 characters. How many passwords are there

   (a) in total?

   $62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 = 62^8 \approx 2.18 \times 10^{14}$

   ↑

   case sensitive = 26 uppercase + 26 lowercase
   alphanumeric = ↑ ↗ plus 10 digits
   } 62 characters

   (b) that start with a lower case letter?

   $26 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 = 26 \times 62^7 = 9.156 \times 10^{13}$

   (c) that start with a lower case letter and end in a number?

   $26 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 10 = 26 \times 62^6 \times 10 = 1.4768 \times 10^{13}$

2. (9 points) Two hundred families in Greater Victoria were surveyed to find out what kind of pet they own. The most common pets were dogs and cats, and the associated results are shown below.

	dog	no dog	
cat	24	70	94
no cat	40	66	106
	64	136	200

In your answers below, show enough work that I can see which method you are using.

(a) What's the probability that a family chosen at random owns a dog?

$$P(D) = \frac{n(D)}{n_{tot}} = \frac{64}{200} = \boxed{32\%}$$

(b) What's the probability that a family chosen at random owns both a dog and a cat?

$$P(DC) = \frac{n(DC)}{n_{tot}} = \frac{24}{200} = \boxed{12\%}$$

(c) What's the probability that if a family chosen at random owns a dog, that they also own a cat?

$$P(C|D) = \frac{n(DC)}{n(D)} = \frac{24}{64} = \boxed{37.5\%}$$

(d) Are the events "owning a dog" and "owning a cat" independent? Explain briefly, being sure to state the values of the probabilities you are comparing.

$P(C) = \frac{n(C)}{n_{tot}} = \frac{94}{200} = 47\%$

$P(C|D) = 37.5\%$ part (c)

since $P(C) \neq P(C|D)$, dependent

or

$P(D) = 32\%$ from (a)

$P(D|C) = \frac{n(CD)}{n(C)} = \frac{24}{94} = 25.5\%$

since $P(D) \neq P(D|C)$, dependent

3. (8 points) The Saanich city council has four members: Alex, Barbara, Charlie, and Dorothy. Two of these members are to be selected to form a subcommittee to study the citys traffic problems. Assume that there is an equal chance of selecting each council member, that the order of selection doesn't matter, and you cannot select a single person twice in a row.

   (a) How many different subcommittees are possible? Explain briefly.

   AB BC CD
   AC BD
   AD

   6 committees (from sample space)

   (b) What is the probability that Dorothy is a member of the committee?

   $P(D) = \frac{n(D)}{n_{tot}} = \frac{3}{6} =$ ½ or 50%

   (c) What is the probability that Charlie and Dorothy are both selected?

   $P(CD) = \frac{n(CD)}{n_{tot}} =$ $\frac{1}{6}$ or $16.\bar{6}\%$

   (d) What is the probability either Charlie or Dorothy or both are selected? Show enough work that I can tell what method you are using.

   $P(C \text{ or } D) = \frac{n(C \text{ or } D)}{n_{tot}} = \frac{5}{6} = 83.\bar{3}\%$

   (e) Calculate your answer to part (d) again, but use a different method and show your work!

$P(C \text{ or } D) = 1 - P(\overline{C \text{ or } D})$
$= 1 - P(AB)$
$= 1 - 1/6$
$= 5/6$

or

$P(C \text{ or } D) = P(C) + P(D) - P(CD)$
$= 1/2 + 1/2 - 1/6$
$= 5/6$

4. (7 points) The response time for a fire department is the time from when a 911 call was made until the time at which the firefighters show up at the scene of the fire. In Calgary, the average response time is 7.1 minutes, with a standard deviation of 1.6 minutes. Assume that the response time is normally distributed.

   (a) What is the probability that the response time for a randomly chosen call is between 5 and 10 minutes?

area = 0.4049

area = 0.4649

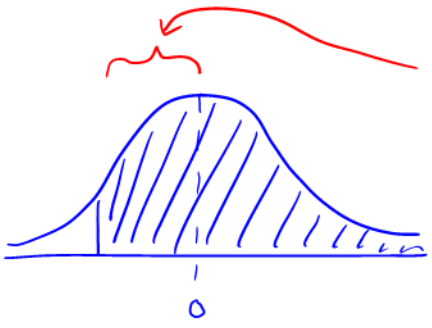
$z_{low} = \frac{x-\mu}{\sigma} = \frac{5-7.1}{1.6} \approx -1.3125 \approx -1.31$

$z_{high} = \frac{x-\mu}{\sigma} = \frac{10-7.1}{1.6} \approx 1.8125 \approx 1.81$

$P = 0.4049 + 0.4649$

$= 0.8698$ so 87% (87.0% also acceptable)

   (b) 85% of calls have a response time that is larger than a certain value. Find that value.



so this area = 0.85 - 0.5 = 0.35, which has

z	0.04
1.0	0.3508

and z is negative because it is to the left of the mean at 0.

So $z = -1.04$

$z = \frac{x-\mu}{\sigma}$

$\sigma z = x - \mu$

$\mu + \sigma z = x$

$x = 7.1 + 1.6(-1.04)$

$= 5.436$

≈ 5.4 minutes