STAT 157 – Practice Test 2

November 29, 2017 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 30 points

1. (7 points) A computer system requires an eight-character alphanumeric password.

   (a) How many different passwords are possible if the passwords are not case-sensitive?

multiplication rule: 36 36 36 36 36 36 36 36

26 letters plus 10 digits

$\boxed{= 36^8 \text{ or } 2.82 \times 10^{12}}$

2

(2 821 109 907 456 if you insist)

   (b) How many different passwords are possible if the passwords are case-sensitive?

multiplication rule: 62 _ _ _ _ _ _ _

2

26 uppercase letters
26 lowercase
10 digits

$\boxed{= 62^8 \text{ or } 2.18 \times 10^{14}}$

(218 340 105 584 896 if you insist)

   (c) How many different passwords are possible if the passwords are case-sensitive and must contain at least one capital letter?

total passwords: 62^8 from part (b)

illegal passwords = no capital letters

$= 36^8$

3

legal passwords = total − illegal

$= 62^8 - 36^8 \approx 2.17 \times 10^{14}$

2. (8 points) Of the three hundred consumers who bought a new Mazda3 at the Pacific Mazda car dealership last year, two hundred of them bought a sedan while the rest bought the hatchback. Fifty of the sedans had a standard transmission, while seventy-five of the hatchbacks did.

   (a) Complete the contingency table below using the above information.

	standard	automatic	
sedan	50	150	200
hatchback	75	25	100
	125	175	

   (b) Calculate the probability that a random customer bought a hatchback with automatic transmission.

$$P(HA) = \frac{n(HA)}{n_{tot}} = \frac{25}{300} = 0.08\overline{3} \quad \text{or} \quad \boxed{8.\overline{3}\,\%}$$

(1/12 if you prefer)

   (c) Calculate the probability that if a random customer bought a hatchback, that it also had an automatic transmission.

$$P(A|H) = \frac{n(AH)}{n(H)} = \frac{25}{100} = 0.25 \quad \text{or} \quad \boxed{25\,\%}$$

(or 1/4)

   (d) Calculate the probability that a random customer bought a hatchback or chose an automatic transmission or both.

$$P(A \text{ or } H) = \frac{n(A \text{ or } H)}{n_{tot}} = \frac{130 + 25 + 75}{300} = \frac{250}{300} = 5/6$$

$$\boxed{= 83.\overline{3}\,\%}$$

$$\underline{\text{or}} \quad P(A \text{ or } H) = \frac{n(A) + n(H) - n(AH)}{n_{tot}}$$

$$= \frac{175 + 100 - 25}{300} = \text{same result}$$

3. (7 points) Three runners run a 100-metre sprint, and the order in which they finish is recorded. The runners' names are Ali, Bob, and Charles. Let's assume that all runners are equally qualified, so that all possible outcomes are equally likely.

   For the following questions, please show enough work that I can see what method you are using.

   (a) Write out the sample space for this situation.

   ABC BAC CAB
   ACB BCA CBA

   6 outcomes

   (b) What is the probability that Ali wins the race?

   $$P(A \text{ first}) = \frac{n(A \text{ first})}{n_{tot}} = \frac{2}{6} = \frac{1}{3} \text{ or } \boxed{33\%}$$

   (c) What is the probability that Ali comes first or second?

   $$P(A \text{ first or second}) = \frac{n(A \text{ first or second})}{n_{tot}} = 4/6 = 2/3 \text{ or } \boxed{67\%}$$

   (d) Are "Ali coming first" and "Bob coming second" independent? Explain, including values of appropriate probabilities.

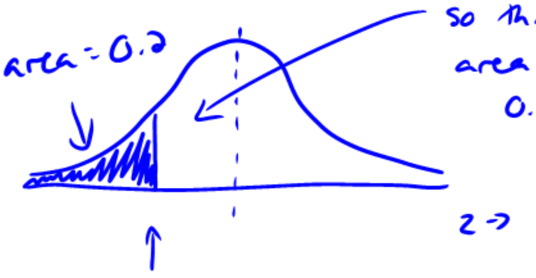
   $$P(A \text{ first} \mid B \text{ second}) = \frac{n(A \text{ first } \underline{\text{and}} \text{ } B \text{ second})}{n(B \text{ second})} = \frac{1}{2} = 50\%$$

   $P(A \text{ first}) = 33\%$ from part (b)

   since these probabilities are not equal, events are dependent (not independent)

4. (8 points) The mayor of Victoria was informed that household water usage is a normally distributed random variable with mean of 25 gallons/day and a standard deviation of 6 gallons/day.

   (a) If the mayor wants to give a tax rebate to the lowest 20% of water users, what should the gallons/day cutoff be?



from normal table

$z = \frac{x-\mu}{\sigma}$

$z\sigma = x - \mu$

$x = \mu + z\sigma$

$= 25 + (-0.84)(6)$

$= 19.96$

$\boxed{\approx 20 \text{ gallons/day}}$

5

(-1) if used 0.2 area on normal table

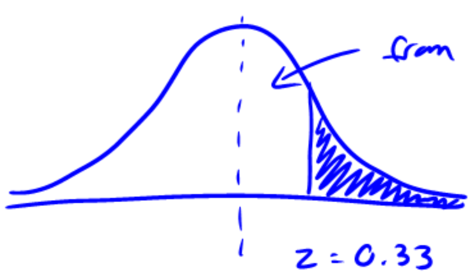
(-1) if forgot negative sign

(-½) if no units

   (b) Calculate the probability that a randomly-chosen household will use more than 27 gallons per day.

$z = \frac{x-\mu}{\sigma} = \frac{27-25}{6} = 0.3\overline{3}$

3



$P(z > 0.33) = 0.5 - 0.1293$

$= 0.3707$

or 37%