Stat 254 – Continuous Probability Distributions

1. Suppose that some phenomenon has the following probability distribution.

$$f(x) = \begin{cases} kx^2 & \text{for } 6 \le x \le 12 \\ 0 & \text{otherwise} \end{cases}$$

   a) Calculate k so that $f(x)$ is indeed a probability distribution function.
   b) Calculate the probability of x being between 6 and 9.
   c) Calculate the mean value of x (also called the expectation value of x).

2. Suppose you have a continuous variable with the probability distribution below.

$$f(x) = \begin{cases} \frac{k}{x} & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   a) Calculate k so that $f(x)$ is indeed a probability distribution function.
   b) Calculate μ, the mean value of x.
   c) Calculate the standard deviation σ of x.

3. Suppose that some variable has the following probability distribution

$$f(x) = \begin{cases} 5e^{-5x} & \text{for } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

   a) Calculate the probability that x will be between 1 and 3.
   b) Calculate the probability that x will be less than 0.5.
   c) From your answer to (b), calculate the probability that x will be greater than 0.5.

4. Suppose that some random variable has the probability distribution given by

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2 - x & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   Show that this truly represents a probability distribution function by verifying that the area under the curve is exactly 1.

Answers

1. a) $k = \frac{1}{504}$ b) 34% c) 9.6

2. a) $k = \frac{1}{\ln 2} \approx 1.4$ b) $\mu = k = \frac{1}{\ln 2} \approx 1.4$ c) $\sigma \approx 0.29$

3. a) 0.67% b) 92% c) 8.2%

4. see solution on web