Stat 254: Section 4-3 Exponential Exercises

1. Let T equal the time in minutes between two successive arrivals at the drive-up window of the local Tim Hortons. If T has an exponential distribution with $k = 0.5$,
   (a) find the expected time between two successive arrivals.
   (b) find the standard deviation of the time between successive arrivals.

2. Testing of fans commonly used in a certain type of desktop computer suggests that the time until failure can be modeled by an exponential distibution with a mean time until failure of 45000 hours. What is the probability that a randomly selected fan will last
   (a) at least 40000 hours?
   (b) at most 50000 hours?
   (c) between 40000 and 50000 hours?

3. The time between clicks of a Geiger counter (counting radioactivity) can be modeled by an exponential distribution. Let's say that when measuring a certain area, the mean time between clicks for a Geiger counter is 6 seconds. There is a 75% probability that the time between the next two clicks is longer than t. Calculate t.

4. Suppose that the amount of time a student spends on a statistics problem is exponentially distibuted with mean μ. What is the probability that the time a student spends on a particular problem is
   (a) longer than the mean time μ? Give an exact answer.
   (b) exactly equal to the mean time? (Assume that you can measure the time to infinite precision.)

Solutions

1. (a) $\mu = 1/k = 1/0.5 = 2$ minutes

   (b)
$$\begin{aligned}
\sigma^2 &= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 \\
&= \int_0^{\infty} x^2\, e^{-kx}\,dx - \frac{1}{k^2} \\
&= \frac{-e^{-kx}}{k^2}(k^2x^2 + 2kx + 2)\Big|_0^{\infty} - \frac{1}{k^2} \\
&= \frac{2}{k^2} - \frac{1}{k^2} \\
&= \frac{1}{k^2}
\end{aligned}$$
   so $\sigma = \dfrac{1}{k} = 2$ minutes, and yes, it's equal to the mean

2. Since $\mu = 45000$, then $k = \dfrac{1}{45000}$

   (a) $P(x > 40000) = \int_{-\infty}^{\infty} f(x)\,dx = \int_{40000}^{\infty} e^{-kx}dx = -e^{-kx}\big|_{40000}^{\infty} = -e^{-\frac{40000}{45000}}$
   $\approx 0.411111 \approx 41\%$

   (b) $P(x < 50000) = \int_0^{50000} e^{-kx}dx = -e^{-kx}\big|_0^{50000} = -e^{-\frac{50000}{45000}} + 1 \approx 0.67081 \approx 67\%$

   (c) $P(40000 < x < 50000) = \int_{40000}^{50000} e^{-kx}dx = -e^{-kx}\big|_{40000}^{50000} = -e^{-\frac{50000}{45000}} + e^{-\frac{40000}{45000}}$
   $\approx 0.08192 \approx 8\%$

3. Recall that $\mu = 1/k$, so
   $P(x > t) = \int_t^{\infty} f(x)\,dx = \int_t^{\infty} e^{-kx}dx = -e^{-kx}\big|_t^{\infty} = e^{-kt}$
   but $P(x > t) = 0.75$ so $e^{-kt} = 0.75$ and
   $t = -\dfrac{\ln 0.75}{k} = -\mu \ln 0.75 = -6 \ln 0.75 \approx 1.72609 \approx 1.7$ minutes

4. (a) Recall that $\mu = 1/k$, so
   $P(x > 1/k) = \int_{1/k}^{\infty} f(x)\,dx = \int_{1/k}^{\infty} e^{-kx}dx = -e^{-kx}\big|_{1/k}^{\infty} = e^{-1}$

   (b) $P(x = \mu) = 0$