Suggested Problems 26. Large Sample Hypothesis Tests

1. We want to test whether two population means μ_1 and μ_2 are different. State H_0 and H_a.

2. We want to test whether a population proportion is greater than 0.045. State H_0 and H_a.

3. A certain brand of rope is required to have a mean breaking strength of exactly 70 pounds. A sample of 49 ropes reveals $\overline{x} = 69.1$ pounds and a SD of 3.5 pounds. Is there evidence that the requirements are not being met? Test with $\alpha = 0.01$.

4. Is there evidence of a difference in mean fuel efficiencies (miles per gallon)? Test with $\alpha = 0.05$.
Prius (Group 1): $n = 40$ $\overline{x} = 59$ $s = 2$
Smart (Group 2): $n = 40$ $\overline{x} = 60$ $s = 2$

5. At a ball bearing factory a sample of 1200 ball bearings reveals that 6 are defective. Is this evidence that the proportion of defective ball bearings is different than 0.006? Test with $\alpha = 0.05$.

6. Are the proportions of hospital admissions for heart-related ailments the same for males and females? Test with $\alpha = 0.01$.
Males (Group 1): $n = 1000$ $\hat{p} = 0.052$
Females (Group 1): $n = 1000$ $\hat{p} = 0.023$

7. A bottling company wants to test whether the mean fill volume is less than 355 mL. A sample of 32 cans reveals $\overline{x} = 354.4$ mL and $s = 1.9$ mL. Test with $\alpha = 0.02$.

8. A shipping company wants to test whether the proportion of shipped items that are lost in transit is more than 0.004. A sample of 1800 records showed that 17 were lost in transit. Test with $\alpha = 0.05$.

9. An experiment compared two brands of tires. The distance it took each tire to wear out was recorded, in thousands of kilometers. Test whether $\mu_1 > \mu_2$ at the 5% significance level.

Brand	n	$\overline{x}$	s
1	45	38.7	5.1
2	42	36.0	4.8

10. An experiment studied two production lines at a manufacturing plant. A random sample of items from each line was collected and the number of defective items was recorded. Test whether $p_1 < p_2$ at the 1% significance level.

Line	n	number defective
1	350	7
2	800	19

Solutions

(1) $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 \neq 0$

(2) $H_0: p = 0.045$ $H_a: p > 0.045$

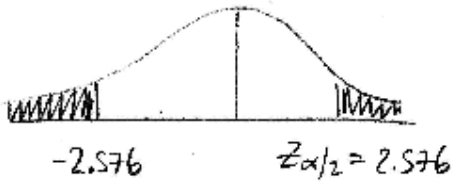
(3) 1) $H_0: \mu = 70$ $H_a: \mu \neq 70$ two-tailed

2) Assumptions: $n \geq 30$ ←

3) $$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$
$$= \frac{69.1 - 70}{(3.5/\sqrt{49})}$$
$$= -1.8$$

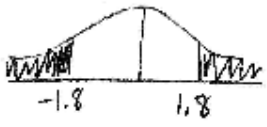
4) Rejection Region
$\alpha = 0.01$

two-tailed



5) Don't reject H_0
$\mu = 70$

6) $p = P(|z| > 1.8)$
$= 2(0.5 - 0.4641)$
$= 0.0718$



(4) 1) $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 \neq 0$

(two-tailed)

2) Assumptions: $n_1 \geq 30$ and $n_2 \geq 30$ ✓

3) $$z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

To find D_0, look at H_0:

$H_0: \mu_1 - \mu_2 = 0$

$D_0 = 0$

$$= \frac{59 - 60 - 0}{\sqrt{\left(\frac{2^2}{40} + \frac{2^2}{40}\right)}}$$

$$\simeq -2.24$$

4) Rejection Region

$\alpha = 0.05$ (two-tailed)

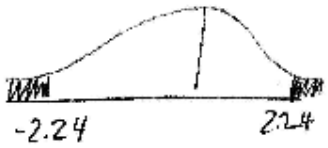
5) Reject H_0

$\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

6) $p = P(|z| > 2.24|)$

$= 2(0.5 - 0.4875)$

$= 0.025$



(5) 1) $H_0: p = 0.006$ $H_a: p \neq 0.006$

(two-tailed)

2) Assumptions: $np_0 = 1200(0.006) > 5$ ✓

$q_0 = 1 - p_0$ $nq_0 = 1200(0.994) > 5$ ✓

3) $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$ $\hat{p} = \dfrac{6}{1200}$

$= \dfrac{\frac{6}{1200} - 0.006}{\sqrt{\frac{0.006(0.994)}{1200}}}$

≈ -0.45

4) Rejection Region (two-tailed)

$\alpha = 0.05$

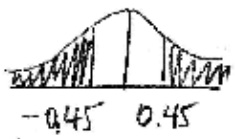
5) Don't reject H_0.

$p = 0.006$

6) $p = P(|z| > 0.45)$

$= 2(0.5 - 0.1736)$

$= 0.6528$



(6) 1) $H_0: p_1 - p_2 = 0$ $\quad H_a: p_1 - p_2 \neq 0$

two-tailed

2) $n_1\hat{p}_1 > 5$, $n_1\hat{q}_1 > 5$, $n_2\hat{p}_2 > 5$, $n_2\hat{q}_2 > 5$ ✓

$\hat{q}_1 = 1 - \hat{p}_1 = 0.948$ $\quad \hat{q}_2 = 1 - \hat{p}_2 = 0.977$

3) $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= \dfrac{0.052 - 0.023}{\sqrt{\frac{75(1925)}{2000^2}\left(\frac{1}{1000} + \frac{1}{1000}\right)}}$

≈ 3.41

$x_1 = n_1\hat{p}_1 = 52$

$x_2 = n_2\hat{p}_2 = 23$

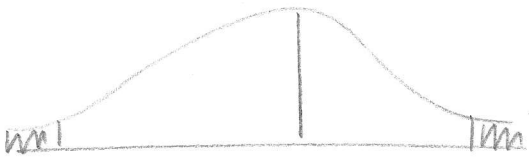
$\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{75}{2000}$

$\hat{q} = 1 - \hat{p} = \dfrac{1925}{2000}$

4) Rejection Region

$\alpha = 0.01$

two-tailed



-2.576 $\quad z_{\alpha/2} = 2.576$

5) Reject H_0

$p_1 - p_2 \neq 0$ or $p_1 \neq p_2$

6) $p = P(|z| > 3.41)$

$= 2(0.5 - 0.4997)$

$= 0.0006$

-3.41 $\quad 3.41$

(7) 1) $H_0: \mu = 355$ $H_a: \mu < 355$

left-tailed

2) $n \geq 30$ ✓

3) $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

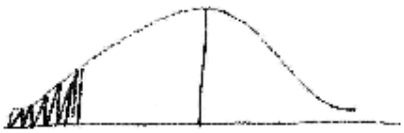
$= \dfrac{354.4 - 355}{1.9/\sqrt{32}}$

≈ -1.79

4) Rejection Region

$\alpha = 0.02$

left-tailed



$-z_\alpha = -2.054$

5) Don't reject H_0

$\mu = 355$

6) $p = P(z < -1.79)$

$= 0.5 - 0.4633$

$= 0.0367$

left-tailed

-1.79

(8) 1) $H_0: p = 0.004$ $H_a: p > 0.004$

right-tailed

2) $np_0 = 1800(0.004) > 5$ ✓

$q_0 = 1 - p_0 = 0.996$

$nq_0 = 1800(0.996) > 5$ ✓

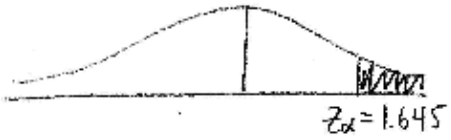
3) $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ $\hat{p} = \frac{17}{1800}$

$= \dfrac{\frac{17}{1800} - 0.004}{\sqrt{\left(\frac{0.004(0.996)}{1800}\right)}}$

≈ 3.66

4) Rejection region right-tailed

$\alpha = 0.05$



5) Reject H_0

$p > 0.004$

6) $p = P(z > 3.66)$

$z = 3.66$ is off the z-table

≈ 0

(9) 1) $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 > 0$ (right-tailed)

2) $n_1 \geq 30$ and $n_2 \geq 30$ ✓

3) $z = \dfrac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

To find D_0, look at H_0: $H_0: \mu_1 - \mu_2 = 0$ → D_0

$= \dfrac{38.7 - 36.0 - 0}{\sqrt{\left(\frac{5.1^2}{45} + \frac{4.8^2}{42}\right)}}$

≈ 2.54

4) Rejection Region (right-tailed)

$\alpha = 0.05$

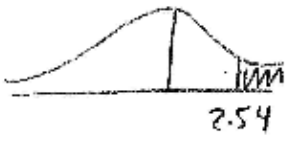
5) Reject H_0

$\mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

6) $p = P(z > 2.54)$

$= 0.5 - 0.4945$

$= 0.0055$



(10) 1) $H_0: p_1 - p_2 = 0$ $H_a: p_1 - p_2 < 0$

(left-tailed)

2) $\hat{p}_1 = \frac{7}{350}$ $\hat{q}_1 = 1 - \hat{p}_1 = \frac{343}{350}$

$\hat{p}_2 = \frac{19}{800}$ $\hat{q}_2 = 1 - \hat{p}_2 = \frac{781}{800}$

$n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, $n_2\hat{q}_2$ all > 5 ✓

3) $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$\hat{p} = \frac{7+19}{350+800} = \frac{26}{1150}$

$\hat{q} = 1 - \hat{p} = \frac{1124}{1150}$

$= \dfrac{\frac{7}{350} - \frac{19}{800}}{\sqrt{\frac{26(1124)}{1150^2}\left(\frac{1}{350} + \frac{1}{800}\right)}}$

≈ -0.39

4) Rejection Region $\alpha = 0.01$ (left-tailed)

$-z_\alpha = -2.326$

5) Don't reject H_0: $p_1 - p_2 = 0$ or $p_1 = p_2$

-0.39

6) $p = P(z < -0.39)$

$= 0.5 - 0.1517$

$= 0.3483$