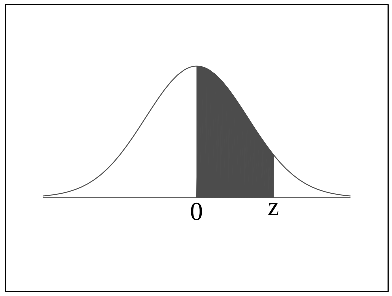
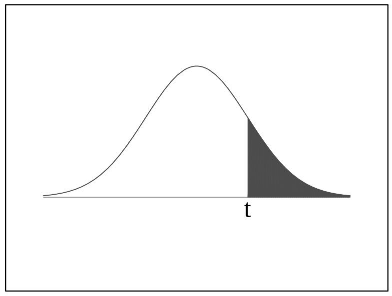
Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

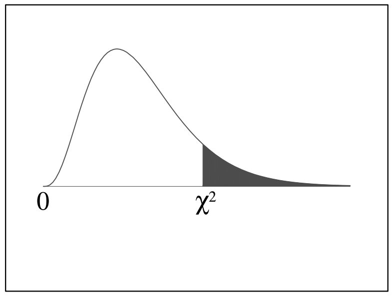
t-Distribution Table



The shaded area is equal to α for $t = t_{\alpha}$.

df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
∞	1.282	1.645	1.960	2.326	2.576

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_\alpha$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Tests of Hypotheses

1. **Test a Population Mean**

   Null Hypothesis: $H_0 \colon \mu = \mu_0$

   Test Statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a \colon \mu > \mu_0$	$z > z_\alpha$ (upper-tailed test)
$H_a \colon \mu < \mu_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a \colon \mu \neq \mu_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   If we don't have a normal population, the sample size should be large i.e., $n \geq 30$. If the population standard deviation σ is not known, we use the sample standard deviation s, in this case we need $n \geq 30$ even if the population is normal.

2. **Test a Population Proportion**

   Null Hypothesis: $H_0 \colon p = p_0$

   Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a \colon p > p_0$	$z > z_\alpha$ (upper-tailed test)
$H_a \colon p < p_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a \colon p \neq p_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   The sample size should be large i.e., $np_0 > 5$ and $nq_0 > 5$.

3. **Test a Difference Between two Population Means**

   Null Hypothesis: $H_0 \colon \mu_1 - \mu_2 = D_0$

   Test Statistic: $z = \dfrac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a \colon \mu_1 - \mu_2 > D_0$	$z > z_\alpha$ (upper-tailed test)
$H_a \colon \mu_1 - \mu_2 < D_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a \colon \mu_1 - \mu_2 \neq D_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   If we don't have normal populations, the sample sizes should be large i.e., $n_1 \geq 30$ and $n_2 \geq 30$, and the two samples should be independently randomly selected. If σ_1 and σ_2 are not known, we use s_1 and s_2, in this case we need $n_1 \geq 30$ and $n_2 \geq 30$.

4. **Test a Difference Between two Population Proportions**

   Null Hypothesis: $H_0 \colon p_1 - p_2 = 0$

   Test Statistic: $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}(1/n_1 + 1/n_2)}}$, where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$.

Alternative Hypothesis	Rejection region at the significance level α
$H_a \colon p_1 - p_2 > 0$	$z > z_\alpha$ (upper-tailed test)
$H_a \colon p_1 - p_2 < 0$	$z < -z_\alpha$ (lower-tailed test)
$H_a \colon p_1 - p_2 \neq 0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   The sample sizes should be large i.e., $n_1\hat{p}_1, n_1\hat{q}_1, n_2\hat{p}_2$, and $n_2\hat{q}_2$ should all be greater than 5, and the two samples should be independently randomly selected.

5. ***p*-value**

   The p-value is the smallest value of α for which H_0 could be rejected. It is the probability that the null hypothesis could produce an observed sample at least as extreme as the one that was observed. The smaller the p-value, the stronger the evidence against H_0.

   For an upper-tailed test, the p-value is $P(Z > z_{\text{obs}})$.

   For a lower-tailed test, the p-value is $P(Z < z_{\text{obs}})$.

   For a two-tailed test, the p-value is $P(Z > |z_{\text{obs}}|) + P(Z < -|z_{\text{obs}}|) = 2\,P(Z > |z_{\text{obs}}|)$.

Inference from Small Samples

The t-distribution is used to make inference about a population mean μ if

1. The population from which the sample is drawn is (approximately) normally distributed.
2. The sample size is small (i.e., $n < 40$).
3. The population standard deviation σ is not known.

The degrees of freedom are $df = n - 1$.

A $(1-\boldsymbol{\alpha})100\%$ **Small-Sample Confidence Interval for** $\boldsymbol{\mu}$ is of the form

$$\overline{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}.$$

Small-Sample Test about a Population Mean

Null Hypothesis: $H_0\colon \mu = \mu_0$

Test Statistic: $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon \mu > \mu_0$	$t > t_\alpha$ (upper-tailed test)
$H_a\colon \mu < \mu_0$	$t < -t_\alpha$ (lower-tailed test)
$H_a\colon \mu \neq \mu_0$	$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$ (two-tailed test)

The t-distribution is used to make inference about $\mu_1 - \mu_2$ if

1. The two populations from which the samples are drawn are (approximately) normally distributed.
2. The samples are small (i.e., $n_1 < 40$ and $n_2 < 40$) and independent.
3. The standard deviation σ_1 and σ_2 of the two populations are unknown but are equal.

The degrees of freedom are $df = n_1 + n_2 - 2$.

A $(\mathbf{1}-\boldsymbol{\alpha})\mathbf{100\%}$ **Small-Sample Confidence Interval for** $\boldsymbol{\mu_1} - \boldsymbol{\mu_2}$ is of the form

$$(\overline{x}_1 - \overline{x}_2) \pm t_{\alpha/2}\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where $s^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$ is the pooled variance for the two samples.

Small-Sample Test about a Difference Between two Population Means

Null Hypothesis: $H_0\colon \mu_1 - \mu_2 = D_0$

Test Statistic: $t = \dfrac{\overline{x}_1 - \overline{x}_2 - D_0}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, where s^2 is the pooled variance.

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon \mu_1 - \mu_2 > D_0$	$t > t_\alpha$ (upper-tailed test)
$H_a\colon \mu_1 - \mu_2 < D_0$	$t < -t_\alpha$ (lower-tailed test)
$H_a\colon \mu_1 - \mu_2 \neq D_0$	$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$ (two-tailed test)

If the two populations do not have equal standard deviations, we do not use the pooled variance. Instead we use the following formulas for standard error and degrees of freedom.

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad \text{and} \quad df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

Chi-Square Distribution

Inference about a Population Variance

If the population from which the sample is selected is (approximately) normally distributed, then

$$\frac{(n-1)s^2}{\sigma^2}$$

has a chi-square distribution with $n-1$ degrees of freedom.

A $(1-\alpha)100\%$ Confidence Interval for σ^2 is of the form

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}.$$

The confidence interval for σ can be obtained by taking the square root of the two limits of the above interval.

Test about a Population Variance

Null Hypothesis: $H_0\colon \sigma^2 = \sigma_0^2$

Test Statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a\colon \sigma^2 > \sigma_0^2$	$\chi^2 > \chi^2_\alpha$ (upper-tailed test)
$H_a\colon \sigma^2 < \sigma_0^2$	$\chi^2 < \chi^2_\alpha$ (lower-tailed test)
$H_a\colon \sigma^2 \neq \sigma_0^2$	$\chi^2 > \chi^2_{\alpha/2}$ or $\chi^2 < \chi^2_{(1-\alpha/2)}$ (two-tailed test)

Goodness-of-fit Test

A **Multinomial Experiment** is an experiment with the following properties.

1. It consists of n identical independent trials.
2. Each trial results in one of k possible outcomes.
3. The probabilities p_i of the possible outcomes remain constant for each trial.

The test statistic for a goodness-of-fit test of a multinomial experiment is

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where:

$O_i =$ observed frequency of category i, and $E_i =$ expected frequency of category $i = np_i$.

In a goodness-of-fit test, the degrees of freedom are $df = k-1$. It is always an upper-tailed test. The number of trials n should be large enough so that $np_i > 5$ for all i.

The p-value is $P(\chi^2 > \chi^2_{\text{obs}})$.