Section 4.1: Answer to Question 4

Wednesday, March 20, 2019 1:46 PM

4. Suppose that some random variable has the probability distribution given by

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2 - x & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Show that this truly represents a probability distribution function by verifying that the area under the curve is exactly 1.

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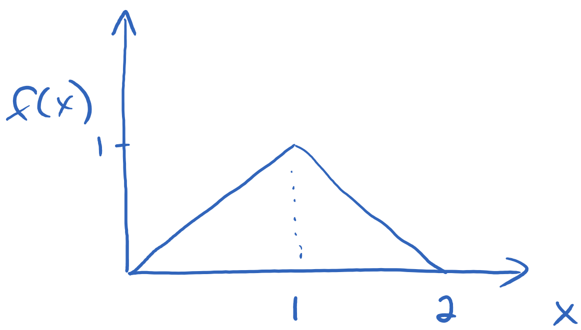

geometric method:

each of these triangles has area

$$A = \tfrac{1}{2} bh = \tfrac{1}{2}(1)(1) = \tfrac{1}{2}$$

so total area $= 2A = 1$ ✓

if you insist,

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 x\,dx + \int_1^2 (2-x)\,dx$$

$$\int_{-\infty} f(x)\,dx = \int_0 x\,dx + \int_1 (2-x)\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \left(2x - \frac{x^2}{2}\right)\Big|_1^2$$

$$= 1/2 + (4-2) - (2 - 1/2)$$

$$= 1/2 + 2 - 3/2$$

$$= 1$$

✓