Section 2.3: Combinations and Permutations

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permutation ≡ an <u>ordered</u> arrangement of r objects chosen <u>without</u> repetition from a group of n objects

notation: ${}_nP_r$ or P_r^n or $P(n, r)$

↑ we'll use

$${}_nP_r = \frac{n!}{(n-r)!}$$

note: $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1$

example: how many 4-digit PINs are there if repetition of digits is not allowed?

method #1: ___ ___ ___ ___ $= 10\cdot9\cdot8\cdot7$ $= 5040$

10 choices, 9 choices, 8, 7

method #2: order matters, no repetition

$${}_{10}P_4 = 5040$$

note: ${}_{10}P_4 = \dfrac{10!}{(10-4)!} = \dfrac{10!}{6!} = \dfrac{10\cdot9\cdot8\cdot7\cdot\cancel{6\cdot5\cdot4\cdot3\cdot2\cdot1}}{\cancel{6\cdot5\cdot4\cdot3\cdot2\cdot1}}$

special case: how many ordered arrangements can we make out of r objects?

$${}_rP_r = \frac{r!}{(r-r)!} = \frac{r!}{0!} = \frac{r!}{1} = r!$$

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combination ≡ an <u>unordered</u> arrangement of r objects chosen <u>without</u> <u>repetition</u> from a group of n objects

examples: poker hands
pizza toppings

notation: ${}_nC_r$ or C^n_r or $C(n, r)$ or $\binom{n}{r}$

↑
we'll use

$${}_nC_r = \frac{n!}{(n-r)!\, r!}$$

why? $${}_nC_r = \frac{{}_nP_r}{r!}$$

example: Pat assigns 10 sample problems to do for practice. Some overworked Bridge students decide they only have time to do 3 of them. How many different groups of problems could they potentially complete?

- order does not matter
- no repetition

$_nC_r = {}_{10}C_3 = 120$

example: The BC Lotto 6/49 draws at random 6 numbers from 49 choices and order of selection does not matter.
(we are ignoring bonus/extra)

When you buy your ticket, how many ways can you choose

a) all 6 winning numbers?
b) 5 of the 6 winning numbers?
c) 4 of the 6 winning numbers?

a) 1

if you want a formal calculation, it's ${}_6C_6 \, {}_{43}C_0$

b) 5 of 6: $\underbrace{{}_6C_5}_{\text{5 of 6 winning numbers}} \cdot \underbrace{{}_{43}C_1}_{\text{1 of 43 losing numbers}} = 258$

c) 4 of 6: ${}_6C_4 \cdot {}_{43}C_2 = 13545$