Section 2.5: Bayes' Rule

Wednesday, January 24, 2018 4:37 PM

Suppose you know $P(A)$, $P(B|A)$, and $P(B|\bar{A})$, but you want to know $P(A|B)$ instead. How do you do it?

- there's a single calculation which I'll detail at the end of the lecture

- but drawing the tree diagram is a much easier approach

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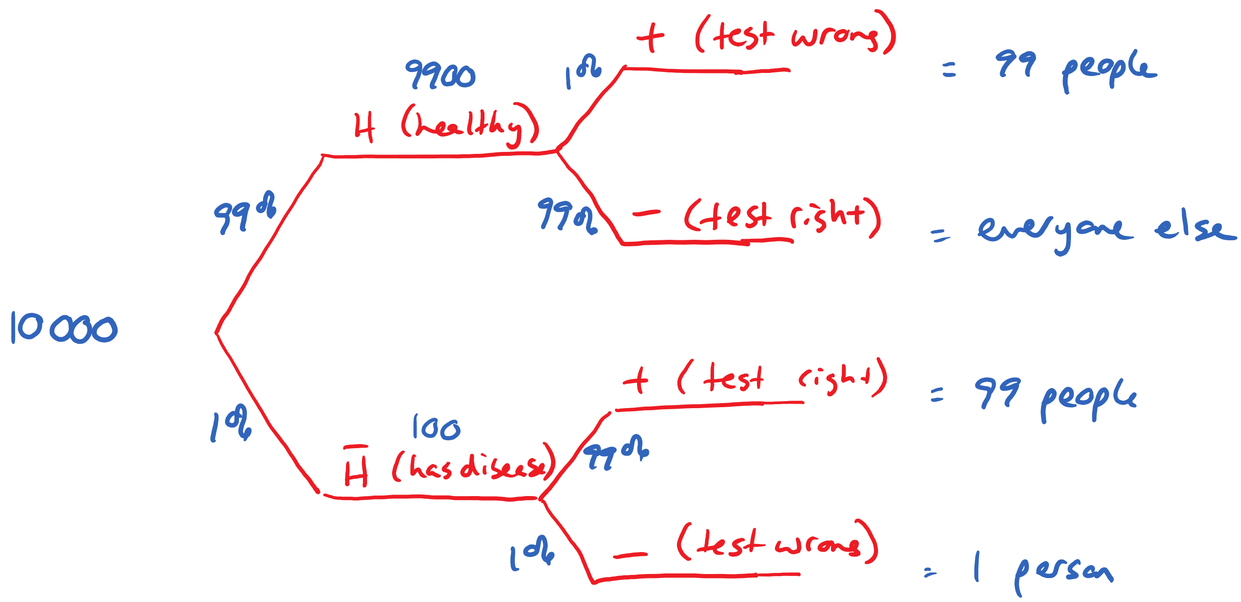
example: The test for a rare disease has 99% reliability. Only one percent of the population has this rare disease.

If the entire population is tested, then some who are healthy will have the test be positive (false positive) and some who have the disease will test negative (false negative).

→ If an individual tests positive, what is the probability that they actually have the disease?

hint: if you prefer, consider the population to be 10,000 individuals and determine the number of individuals in each group

let H = healthy
$\bar{H}$ = has disease
$+$ = test says has disease
$-$ = test says doesn't doesn't have disease



$$P(\bar{H} \mid +) = \frac{n(\bar{H}+)}{n(+)} = \frac{99}{99+99} = \frac{1}{2} \text{ or } 50\%$$

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theory (can omit):

$$P(A \mid B) = \frac{P(AB)}{P(B)} \quad \leftarrow \text{how did we get this?}$$

$$= \frac{P(B \mid A)\,P(A)}{P(B \mid A)\,P(A) + P(B \mid \bar{A})\,P(\bar{A})}$$

and if you insist:

$$P(A_i \mid B) = \frac{P(B \mid A_i)\, P(A_i)}{\sum P(B \mid A_i)\, P(A_i)}$$