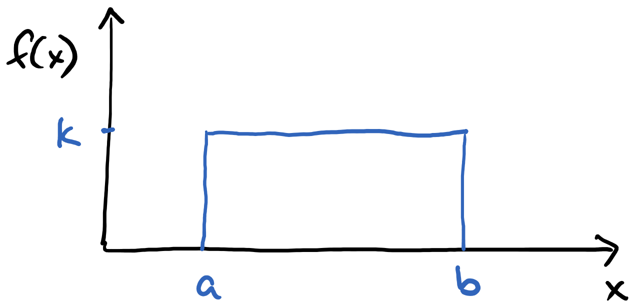
Section 4.2: The Continuous Uniform Probability Distribution

Monday, February 5, 2018 8:31 AM



k = some constant

so, what's the k for f(x) to truly be a probability density function?

$k = \frac{1}{b-a}$ (total area under curve is one)

and continuous uniform case has

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

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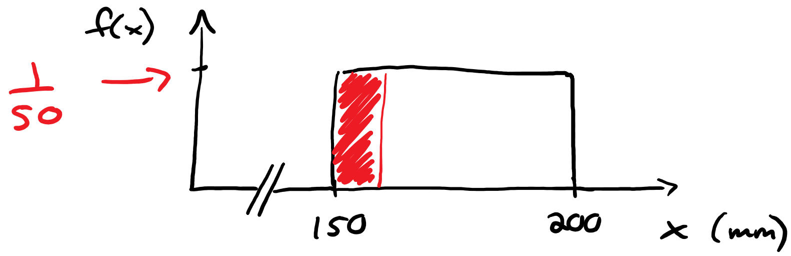
example: suppose the research department of a steel manufacturer believes that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness is a uniform random variable with values between 150 and 200 mm. Any sheets less than 160 mm thick must be scrapped because they are unacceptable to buyers.

a) Calculate the fraction of steel sheets produced by this machine that have to be scrapped.

answer:



$$P(x < 160) = 1/5 \quad \text{(area between 150 + 160)}$$
$$= 20\%$$

b) Calculate the mean and standard deviation of the thickness of sheets produced.

$\mu = 175$ mm by symmetry

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx \quad - \mu^2$$

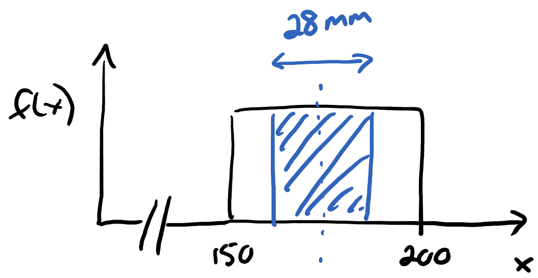
$$= \int_{150}^{200} x^2 \frac{1}{50}\,dx \quad - 175^2$$

$$= \frac{x^3}{150}\Big|_{150}^{200} \quad - 175^2$$

$$= 208.\overline{3}$$

$\sigma = 14.4$ or 14 mm

c) What is the probability that a randomly selected sheet will lie within 1 std dev of the mean? within 2 std dev?



$\mu \pm 1\sigma$: 175 ± 14 mm

$P(161 < x < 189) = \frac{28}{50} = 56\%$

$\mu \pm 2\sigma$: 175 ± 28 mm

$P(147 < x < 203) = 100\%$