Section 3.3: The Poisson Probability Distribution

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Poisson — good model for data that represent the number of occurrances of a specified event in a given unit of time or space

examples:

- number of car accidents at a particular intersection during a given period of time
- number of people standing at a certain street corner at a given time

then X = number of events occurring in a period of time or space

note: X does not have a maximum value → unbounded

so μ = average number of such events expected to occur

and

$$P(X=k) = \frac{\mu^k e^{-\mu}}{k!} \quad \text{where } k=0,1,2,3,\ldots$$

mean: μ

std dev: $\sigma = \sqrt{\mu}$

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example: For a particular cement mix, the average number of cracks per concrete specimen is 2.5. Assume that this number of cracks obeys a Poisson distribution.

a) find the mean and standard deviation
b) what's the probability of having at least one crack in a randomly chosen specimen?

a) $\mu = 2.5$
$\sigma = \sqrt{2.5} = 1.58 \approx 1.6$

b) $P(x \geq 1) = 1 - P(x=0)$

$$P(x=k) = \frac{\mu^k e^{-\mu}}{k!}$$

$$P(x=0) = \frac{(2.5)^0 e^{-2.5}}{0!}$$

$$= 0.082085$$

$$P(x \geq 1) = 1 - 0.082085$$
$$= 0.917915$$
$$\approx 92\%$$

note: $\mu \pm 2\sigma$: the interval from -0.7 to 5.7

and if you sum $\sum_{x=0}^{5} p(x)$, you get 0.958

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example: In nuclear physics, the number of neutrons detected in a particular detector over a certain time period is a Poisson process. What average number of events should you measure so that your uncertainty (standard deviation) is 1% of the mean?

$$\sigma = 0.01\mu$$

$$\sqrt{\mu} = 0.01\mu$$

$$100 = \sqrt{\mu}$$

$$100^2 = \mu$$

$$\mu = 10\,000$$