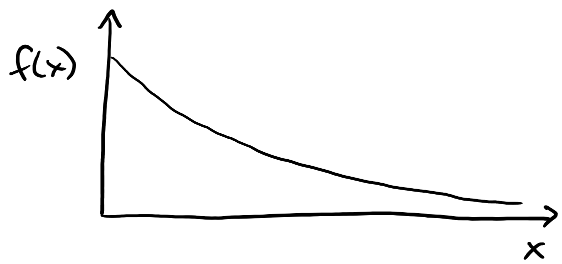
# Section 4.3: The Exponential Distribution

Monday, February 5, 2018 8:57 AM




$$f(x) = \begin{cases} ke^{-kx} & \text{for } x \geq 0 \\ 0 & \text{else where} \end{cases}$$

where $k > 0$

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note: $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} ke^{-kx}\,dx$

$$= \frac{k}{-k} e^{-kx} \Big|_0^{\infty}$$

$$= -e^{-kx} \Big|_0^{\infty}$$

$$= 0 - (-e^0)$$

$$= 1 \quad \checkmark$$

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also:

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

$$= \int_0^{\infty} x\, ke^{-kx}\,dx$$

| D | I |
|---|---|
| $x$ $+$ | $ke^{-kx}$ |

$$= \int_0 x \, k \, e \quad dx$$

| U | | I |
|---|---|---|
| $x$ | $+$ | $ke^{-kx}$ |
| $1$ | $-$ | $-e^{-kx}$ |
| $0$ | | $\frac{1}{k} e^{-kx}$ |

$$= \left(-xe^{-kx} - \frac{1}{k} e^{-kx}\right) \Big|_0^\infty$$

$$= \frac{1}{k} e^{-0} = \frac{1}{k}$$

so if, for example:

$x =$ time between events (random variable)

$\mu =$ mean time between events

$k = \dfrac{1}{\mu} = \dfrac{1}{\text{mean time between events}}$

and $k$ is like a frequency

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example: The time between successive arrivals of trucks at a warehouse has an exponential distribution. On average, three trucks arrive per hour at the warehouse. What are the probabilities that the time between arrivals of successive trucks will be

a) less than 5 minutes?
b) at least 45 minutes?

3 trucks/hour is a frequency

$k =$ 3 trucks per hour

$\mu = \frac{1}{3}$ hours between trucks

(one truck every 20 minutes)