Section 4.3: cont'd

Tuesday, February 6, 2018 3:33 PM

recall from last time:

- three trucks arriving per hour at a warehouse
- time between successive arrivals is exponential

a) want $P(x < 5 \text{ min})$

so μ = average time between arrivals
= 1/3 hour or 20 min

* pick <u>one</u> unit and stick with it

$k = \frac{1}{\mu} = 3$ (trucks/hour if you like units)

$$f(x) = \begin{cases} 3e^{-3x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$P\left(0 < x < \tfrac{1}{12}\right) = \int_a^b f(x)\,dx$$

$$= \int_0^{1/12} 3e^{-3x}\,dx$$

$$-e^{-3x} \Big|^{1/12}$$

$$= -e^{-3x} \Big|_0^{1/12}$$

$$= -e^{-1/4} + e^0$$

$$\approx 0.221199$$

$$\approx 22\%$$

b) want the probability that x will be at least 45 min?

$$P(x > 3/4) = \int_a^b f(x)\,dx$$

$$= \int_{3/4}^{\infty} 3e^{-3x}\,dx$$

$$= -e^{-3x} \Big|_{3/4}^{\infty}$$

$$= 0 + e^{-9/4}$$

$$\approx 0.105399$$
$$\approx 11\%$$

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example: suppose that a system containing a certain type of component has a "time to failure" of the component given by T. The random variable T is modeled nicely by the exponential distribution with a mean time to failure of 5 years.

a) what is the probability that a given component is still functioning after 8 years?

nasty!

b) if five of these components are installed in different systems, what is the probability that at least 2 are functioning at the end of 8 years?

a) $k = \frac{1}{\mu} = \frac{1}{5}$

$$P(T > 8) = \int_8^\infty \frac{1}{5} e^{-t/5}\, dt$$

$$= -e^{-t/5} \Big|_8^\infty$$

$$= 0 + e^{-8/5} \approx 0.201897$$

$$\approx 20\%$$

b) 5 components

lasted / didn't last

P(lasted) = 0.202

binomial

$$P(x=k) = {}_nC_k \; p^k q^{n-k}$$

$$P(x=0) = {}_5C_0 \; (0.202)^0 (0.798)^5$$
$$P(x=1) = {}_5C_1 \; (0.202)^1 (0.798)^4$$

$$P(x \geq 2) = 1 - P(x=0) - P(x=1)$$
$$= 0.266$$

$$\approx 27\%$$