Section 3.9: The Poisson Probability Distribution

Friday, February 1, 2019 3:34 PM

Poisson - good model for data that represent the number of occurrances of a specified event in a given unit of time or space

examples: - the number of car accidents at a particular intersection during a given period of time

- number of pieces of litter in a given area of park at a particular time

then x = number of events occurring in that period of time or region of space

μ = average number of such events expected to occur (also denoted λ)

where

$$P(x=k) = \frac{\mu^k e^{-\mu}}{k!}, \quad \text{where } k = 0, 1, 2, \ldots$$

note: k has no maximum value
→ unbounded at upper end

mean = μ

std dev $\sigma = \sqrt{\mu}$

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example: For a particular cement mix, the average number of cracks per concrete specimen is 2.5.

a) find the mean and standard deviation
b) what's the probability of having at least one crack in a randomly chosen specimen?

a) $\mu = 2.5$ cracks/specimen
$\sigma = \sqrt{\mu} = \sqrt{2.5} \approx 1.58$ or 1.6 cracks/specimen

b) $P(X \geq 1) = 1 - P(X=0)$

$$P(X=k) = \frac{\mu^k e^{-\mu}}{k!}$$

$$P(X=0) = \frac{2.5^0 e^{-2.5}}{0!} \approx 0.082085$$

$$\begin{aligned} P(X \geq 1) &\approx 1 - 0.082085 \\ &\approx 0.917915 \\ &\approx 92\% \end{aligned}$$

note: $\mu \pm 2\sigma$ is the interval from -0.7 to 5.7

and if you sum $\sum_{x=0}^{5} p(x)$, you get 0.958

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example: In nuclear physics, the number of neutrons detected in a particular detector over a certain time period is a Poisson process. What average number of events should you measure so that your uncertainty (standard deviation) is 1% of the mean?

$$\sigma = \sqrt{\mu} = 0.01\ \mu$$

$$100 = \sqrt{\mu}$$

$$\mu = 100^2 = 10000$$

$$100 \times \frac{\sqrt{\mu}}{\sqrt{\mu}} = 0.01\ \frac{\mu}{\sqrt{\mu}} \times 100$$

$$100 = \sqrt{\mu}$$

Test 1 is up to here.