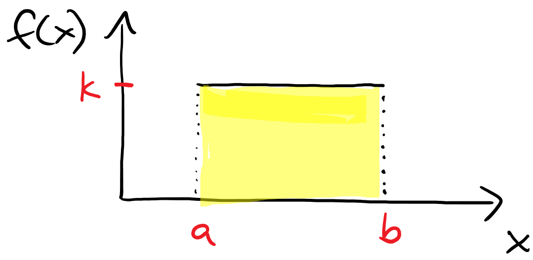
Section 4.2: The Continuous Uniform Probability Distribution

Monday, February 4, 2019 11:43 AM



k = some constant

so, what's the value of k for $f(x)$ to truly be a probability density function?

$$k = \frac{1}{b-a}$$

(area $= k(b-a) = 1$)

continuous uniform case has

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

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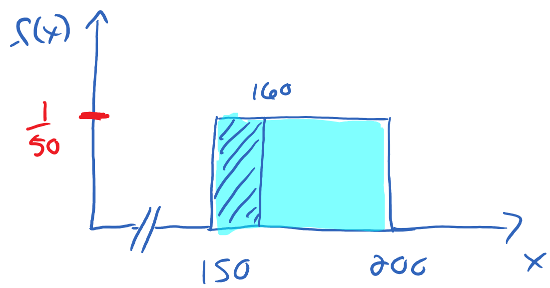
example: suppose the research department of a steel manufacturer believes that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness is a uniform random variable with values between 150 and 200 mm. Any sheets less than 160 mm thick must be scrapped because they are unacceptable to buyers.

a) Calculate the fraction of steel sheets produced by this machine that have to be scrapped.

b) Calculate the mean and std dev of the thickness of sheets produced.

c) What is the probability that a randomly selected sheet will lie within 1 std dev of the mean? within 2 std devs?

a) 

$P(x < 160) = l \cdot w$ (rectangle!)

$= (10)(1/50)$

$= 1/5$ or 20%

b) by symmetry, $\mu = 175$ mm (halfway between 150 & 200mm)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \int_{150}^{200} x^2 \frac{1}{50}\,dx - 175^2$$

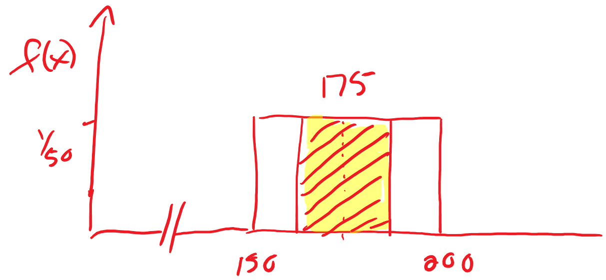
$$= \frac{x^3}{150}\Bigg|_{150}^{200} - 175^2$$

$= 208.\overline{3}$

$\sigma = 14.4$ mm

c) $\mu \pm 1\sigma = 175 \pm 14$ mm

$= [161 \text{ mm}, 189 \text{ mm}]$



$P(161 < x < 189) = \frac{1}{50}(2 \cdot 14)$

$= 56\%$

$\mu \pm 2\sigma = 175 \pm 2(14)$ mm

$P(147 < x < 203) = 100\%$