Section 7.9: Two Samples: Tests on Two Proportions

Thursday, March 28, 2019 3:00 PM

situation: rather than testing a single sample against a claim, we are now testing two samples against each other

so we know $\hat{p}_1$ and $\hat{p}_2$

$H_0: \quad p_1 = p_2 \quad$ or $\quad p_1 - p_2 = 0$

$H_a: \quad p_1 \; \begin{matrix} > \\ < \\ \neq \end{matrix} \; p_2$

test statistic:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$ pooled data

under the conditions

- $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, and $n_2\hat{q}_2$ (yes, all) are all > 5

- the two samples should be independently randomly selected

example: Camosun wants more students to carpool, so now has "carpool-only" parking spaces. To evaluate their effectiveness, personnel monitored samples of randomly selected cars before and after the spaces were implemented, and measured the following data.

	before	after
sample size	1000	1250
carpools	62	93

Has carpooling increased since carpool spaces were implemented? Use a 90% confidence level.

a) state the conditions under which you are performing this test

$n_1 \hat{p}_1 = 62$
$n_1 \hat{q}_1 = 1000 - 62$
$n_2 \hat{p}_2 = 93$
$n_2 \hat{q}_2 = 1250 - 93$

} > 5 ✓

b) state the null and alternate hypotheses

$H_0: \quad p_1 = p_2$ (p_2 = after)

$H_a: \quad p_1 < p_2$ (carpooling has increased)

c) compute the test statistic

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

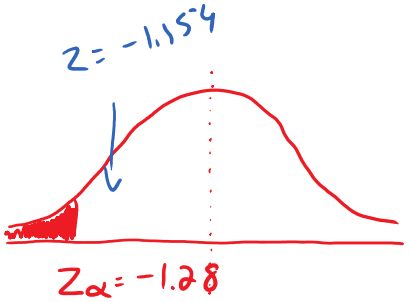
$$= \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

$$= \frac{62 + 93}{1000 + 1250}$$

$$= 0.06\bar{8}$$

$$= \frac{\frac{62}{1000} - \frac{93}{1250}}{\sqrt{(0.06\bar{8})(1 - 0.06\bar{8})\left(\frac{1}{1000} + \frac{1}{1250}\right)}}$$

$$= -1.154$$



z is in the acceptance region

Conclusion: the evidence does not support an increase in carpooling at the 90% level