

STAT 254 – Test 2

Practice Test

Name: Solution Set

Instructor: Patricia Wrean

Total: 25 points

1. (3 points) The manager for a large hotel chain wishes to survey guests who stayed at hotels in that chain within the last year. For the following situations, identify the sampling plan used to pick these guests.

(a) The manager randomly selects 10 hotels and surveys all guests who stayed at those hotels within the last year.

cluster

(b) The manager makes a list of all of the guests who stayed at any of the chain's hotels within the past year and randomly chooses a certain number of guests from that list.

simple random

(c) For each hotel in the chain, the manager chooses a random selection of guests who stayed within the last year.

stratified

2. (2 points) You are shopping for produce at your local grocery store.

(a) At the supermarket, you pick up a package of blueberries, and note that there are about 100 blueberries per package. The produce clerk is a fan of statistics and tells you that the weights of the packages of blueberries are normally distributed. What does this tell you about the distribution of weights for the individual blueberries? (Circle one.)

(i) They must be normally distributed.

(ii) They must be skewed.

(iii) They could have any distribution at all.

since $n \geq 30$, any distribution of weights will have \bar{x} normally distributed

(b) Now you pick up a package of peaches, and each package contains exactly three peaches. The same clerk tells you that the weights of these packages are also normally distributed. What does that tell you about the distribution of weights for the individual peaches? Circle one.

(i) They must be normally distributed.

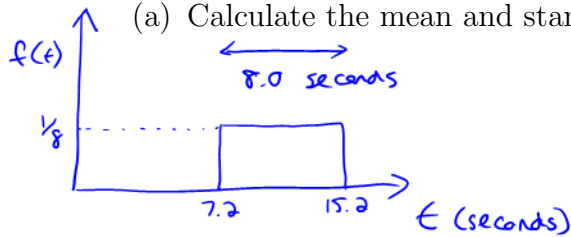
(ii) They must be skewed.

(iii) They could have any distribution at all.

since $n \ll 30$, peaches must be normally distributed to begin with for \bar{x} to be normally distributed

3. (5 points) The boot time for a Microsoft Surface Pro 4 (I5) is known to be uniformly distributed between 7.2 and 15.2 seconds.

(a) Calculate the mean and standard deviation for the boot time for this device.



total area underneath curve is one, so height must be $\frac{1}{8.0}$

(1)

by symmetry, $\mu = \text{midpoint} = \frac{7.2 + 15.2}{2} = 11.2 \text{ seconds}$

(1)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$$

$$= \int_{7.2}^{15.2} \frac{1}{8} x^2 dx - 11.2^2$$

$$= \frac{1}{24} x^3 \Big|_{7.2}^{15.2} - 11.2^2$$

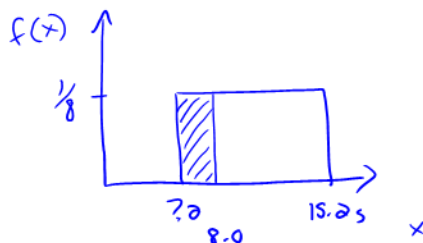
$$= \frac{1}{24} (15.2^3 - 7.2^3) - 11.2^2$$

$$= 5.3$$

(2)

$$\sigma = \sqrt{\sigma^2} \approx 2.3094 \approx 2.3 \text{ seconds}$$

(b) Calculate the probability that the boot time takes less than 8.0 seconds.



$P(x < 8) = \text{shaded area}$

$$= \frac{1}{8} (8.0 - 7.2)$$

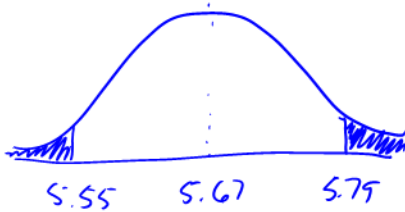
$$= \frac{1}{8} (0.8)$$

$$= 0.1 \quad \text{or} \quad 10\%$$

(1)

4. (6 points) Quarters have masses that are normally distributed with a mean of 5.67 g and a standard deviation of 0.070 g.

- (a) If a vending machine is adjusted to reject quarters whose mass is less than 5.55 g or more than 5.79 g, what is the probability that a legal quarter will be rejected?



area = 0.4564

$$z_{\text{high}} = \frac{x - \mu}{\sigma} = \frac{5.79 - 5.67}{0.070} = 1.714$$

$$z_{\text{low}} = \dots = \frac{5.55 - 5.67}{0.070} = -1.714$$

$$P(\text{rejected}) = 2(0.5 - 0.4564) = 0.0872 = \boxed{9\%}$$

- (b) Find the range of accepted quarters if the machine is readjusted so that the lightest 1.5% and the heaviest 1.5% are rejected.



so this
area = 0.485
with $z = 2.17$

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma = 5.67 \pm 2.17(0.070)$$

$$= 5.5181 \text{ to } 5.8219$$

$$= \boxed{5.52 \text{ to } 5.82 \text{ g}}$$

- (c) If you find that you have 100 quarters in your piggy bank at home, do you think that it's likely that the average mass of these quarters is more than 5.79 g? Explain your answer.

$$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{5.79 - 5.67}{0.070/\sqrt{100}} = 17.14$$

having a z-score this high is extremely unlikely, so $\boxed{\text{No}}$

5. (5 points) A dentist is interested in finding out what fraction of appointments in her practice are cancelled. Her records for the past month indicate that 15 out of the 132 appointments were cancelled.

- (a) Calculate a 90% confidence level for the proportion of cancellations for this dentist's appointments.

$$p = \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \quad (1)$$

$$= \frac{15}{132} \pm 1.645 \sqrt{\left(\frac{15}{132}\right)\left(\frac{117}{132}\right)\left(\frac{1}{132}\right)}$$

$$= 0.113636 \pm 0.045441$$

$$= 0.068196 \text{ to } 0.159077$$

90% CI: $\boxed{6.8\% \text{ to } 15.9\%}$
 or $\boxed{7\% \text{ to } 16\%}$ (1)

check:

$$\left. \begin{array}{l} np = 15 \\ nq = 132 - 15 \end{array} \right\} > 5 \checkmark \quad (1)$$

$$Z_{\alpha/2} = 1.645 \text{ for } 90\% \quad (1)$$

- (b) A trade publication claims that 15% of all dental appointments are cancelled. Is this consistent with the data collected by the dentist? Explain briefly.

Yes, because 15% is contained within the confidence interval. (1)

6. (4 points) A BC fishing company is commissioning a research study to find the mean mass of Chinook salmon caught by its fishing boats. Previous studies have indicated that the standard deviation for this mass is 0.98 kg. How large a sample should be taken to estimate the true mean mass for a catch of this type of salmon to within 0.1 kg with 98% confidence?

$$\mu = \bar{x} \pm \underbrace{z_{\alpha/2} \frac{\sigma}{\sqrt{n}}}_{\text{MOE}}$$

went $\text{MOE} \leq \beta$ where $\beta = 0.1$

$$z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \beta$$

$$\frac{z_{\alpha/2} \sigma}{\beta} \leq \sqrt{n}$$

$$n \geq \left(\frac{z_{\alpha/2} \sigma}{\beta} \right)^2$$

$$\geq \left(\frac{2.326 \cdot 0.98}{0.1} \right)^2 \quad *$$

$$\geq 519.603$$

$$\geq 520$$

(I'd say 525 or 530, myself)

if you used $z_{\alpha/2} = 2.33$, you'll

get $n \geq 521.392$

$n \geq 522$, also acceptable