STAT 254 – Test 1

February 8, 2018 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 30 points

1. (3 points) For Pat's statistics unit in Stat 254, she handed out small boxes of paperclips and asked students to count the number of paperclips in each box.
   (a) What is the experimental unit? box of paperclips
   (b) Name the variable being measured. number of paperclips
   (c) For your answer to part (b), is it
      (i) qualitative
      (ii) quantitative and discrete
      (iii) quantitative and continuous

2. (5 points) A small warehouse employs a supervisor at \$1200 a week, an inventory manager at \$700 per week, six stock workers at \$400 per week, and four drivers at \$500 a week.
   (a) Find the mean and median wage.

   data set: 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 7, 12 (in \$100s per week)

   $$\text{mean} = \frac{\text{sum}}{\text{number}} = \frac{6 \cdot 400 + 4 \cdot 500 + 700 + 1200}{12} = \frac{6300}{12} = \$525$$

   median = \$450

   (b) How many employees make more than the mean wage?

   2

   (c) Which measure of centre best describes a typical wage at this company? Explain briefly.

   the median is better because the mean is pulled towards the outliers at the high end of the data set

3. (2 points) A set of data has a mean of 75 and a standard deviation of 5. What does Tchebysheff's theorem say about the proportion of measurements that fall between 70 and 80?

   (a) It could be zero.

   (b) At least 75% of the measurements will lie between 70 and 80.

   (c) Exactly 75% of the measurements will lie between 70 and 80.

   (d) Approximately 75% of the measurements will lie between 70 and 80.

   After looking at the distribution, you know that this is a more-or-less symmetric, mound-shaped distribution. What can you say about the proportion of measurements that fall between 70 and 80?

   (a) It could be zero.

   (b) At least 68% of the measurements will lie between 70 and 80.

   (c) Exactly 68% of the measurements will lie between 70 and 80.

   (d) Approximately 68% of the measurements will lie between 70 and 80.

4. (3 points) A smoke-detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0.95; by device B, 0.98; and by both devices, 0.94.

   (a) Calculate the probability that if smoke is present, the smoke will not be detected.

method #1

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$
$$= 0.95 + 0.98 - 0.94$$
$$= 0.99$$
$$P(\overline{A \text{ or } B}) = 1 - 0.99 = \boxed{0.01}$$

2

method #2:

	A	$\bar{A}$	
B	0.94	0.04	0.98
$\bar{B}$	0.01	0.01	0.02
	0.95	0.05	

$P(\text{not detected}) \boxed{= 0.01}$

   (b) Calculate the probability that device A detects the smoke but B does not.

method #1

$$P(A) = P(AB) + P(A\bar{B})$$
$$0.95 = 0.94 + P(A\bar{B})$$
$$P(A\bar{B}) = 0.01$$

1

method #2:

from table above

$$P(A\bar{B}) = 0.01$$

5. (2 points) Consider events A and B, with $P(A) = 0.4$ and $P(B) = 0.8$.

   (a) Can A and B be mutually exclusive? Explain briefly.

No.
$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$
$$= 0.4 + 0.8 \underbrace{- P(AB)}_{\text{must be non-zero}}$$
must be non-zero so LHS is ≤ 1

1

   (b) Can A and B be complements (i.e. is it possible that $A = \overline{B}$)? Explain briefly.

1

No $P(A) \neq 1 - P(B)$

6. (4 points) Your engineering company is considering competing for a certain contract. The cost of competing for this contract is \$25 000. You estimate that your bid has a 45% probability of success, which will mean a profit of \$150 000.

   (a) Calculate the expected earnings.

x	$p(x)$
-25000	0.55
$150000 - 25000$	0.45

2

$$E(x) = \sum x\, p(x)$$
$$= (-25000)(0.55) + (125000)(0.45)$$
$$\boxed{= \$42500}$$

[also acceptable: \$ 53750 if assumed \$150000 was net, not gross)

   (b) Your company does not like to bid on risky projects in which the standard deviation of the earnings is more than \$60 000. Should your firm bid on this contract? Explain your reasoning.

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$
$$= (-25000)^2(0.55) + (125000)^2(0.45) -$$

2

$$\sigma = \$74624.1 \quad \boxed{= \$74600}$$

No, the standard deviation is higher than the company's guideline.

note: if calculated
$\sigma^2 = -25000^2(0.55)$ instead,
get \$ 69865.90 instead $-\frac{1}{2}$

if assumed \$150000 was net,
then $\sigma = \$87061.4$
(sign error gives \$ 83019.2 $-\frac{1}{2}$)

7. (6 points) A researcher for BC Ferries randomly selects a sample of sailings from the Vancouver - Victoria route and records whether the sailing departed on time or not and whether it was full. The results are displayed below.

		$\bar{L}$	L	
		on time	late	
F	full	12	3	15
$\bar{F}$	not full	76	9	85
		88	12	

In your answers below, show enough work that I can see which method you are using.

(a) What's the probability that a random sailing was full?

$$P(F) = \frac{n(F)}{n_{tot}} = \frac{15}{100} = 15\%$$

2

(b) What's the probability that a random full sailing was late?

$$P(L|F) = \frac{n(LF)}{n(F)} = \frac{3}{15} = 20\%$$

2

$P(\text{late}) = 12\%$ -1

$P(\text{full} \mid \text{late}) = 3/12 = 25\%$

-1

(c) Are the events "full sailing" and "late departure' independent? Explain briefly, being sure to state the values of the probabilities you are comparing. (Points will only be given if your explanation is valid.)

method #1:

$P(L) = \frac{n(L)}{n_{tot}} = \frac{12}{100} = 12\%$

$P(L|F) = 20\%$ from above

not equal ∴ dependent

2

method #2:

$P(F) = 15\%$ from above

$P(F|L) = \frac{n(FL)}{n(L)} = \frac{3}{12} = 25\%$

not equal ∴ dependent

method #3

$P(LF) = \frac{n(LF)}{n_{tot}} = \frac{3}{100} = 0.03$

$P(L)P(F) = (0.12)(0.15) = 0.018$

not equal ∴ same conclusion

8. (5 points) Let x equal the number of flaws per square meter in a bolt of cloth made in textile manufacturing.

   (a) What is the name of the probability distribution that would best describe x? Explain briefly.

   1

   Poisson - it's the number of events in given time or space

   1 area, in this case

   (b) If the probability that there are no flaws in a square meter of cloth is equal to 16.5%, calculate the mean and standard deviation of x.

$$P(x=k) = \frac{\mu^k e^{-\mu}}{k!}$$

$$P(x=0) = \frac{\mu^0 e^{-\mu}}{0!}$$

$$0.165 = e^{-\mu}$$

$$\ln 0.165 = -\mu$$

$$\mu = -\ln 0.165 \approx 1.80181$$

$$\boxed{\approx 1.80 \text{ flaws/m}^2}$$ 2

$$\sigma = \sqrt{\mu} \approx 1.34232 \boxed{\approx 1.34 \text{ flaws/m}^2}$$ 1

no units $-\frac{1}{2}$

16.5% left as 16.5, not converted to decimal -1