STAT 254 – Test 1

February 8, 2017 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 30 points

1. (4 points) According to the Globe and Mail, an energy analyst in Calgary is studying the weekly energy consumption of Canadians to determine whether moving Daylight Saving Time three weeks earlier is an effective method to save energy. For the purposes of this problem, let's assume that her first preliminary study measures the weekly energy consumption of all residents of Calgary for the past two months.

   (a) What is the population of interest to the analyst?

   all Canadians

   (b) What is the sample used in the study?

   all residents of Calgary

   (c) What is the variable being measured?

   weekly energy consumption

   (d) What type of variable is it? Choose one.

   i qualitative
   ii quantitative, discrete
   iii quantitative, continuous
   iv none of the above

   note: I accepted ii) since students reminded me that electricity and gas meters round to a certain precision

2. (3 points) The number of days off per year for a sample of individuals selected from ~~nine~~ ten different countries is shown below.

   20 23 24 26 30 31 35 36 40 42
   20, 26, 40, 36, 23, 42, 35, 24, 30, 31

   State the mean, median, and range of this data set.

   mean: 30.7

   median: 30.5

   range: 42 - 20 22

   (-½) if you give 42 - 20 instead of single number

3. (3 points) A set of data has a mean of 42 and a standard deviation of 8. What can you say about the proportion of measurements that lie above 58,

   (a) if you know absolutely nothing about the shape of the distribution?

   58 is 2 std dev above the mean

   Tcheby says $\geq 75\%$ are within 2 std of mean so

   $\boxed{\leq 25\% \text{ are above } 58}$

   (b) if you know that the distribution is bimodal but perfectly symmetrical?

   $\boxed{\leq 12.5\% \text{ are above } 58}$

   (c) if instead you know that the distribution is unimodal and beautifully symmetrical?

   can now use Empirical rule, so $\sim 95\%$ are within 2 std dev of mean

   $\boxed{\sim 2.5\% \text{ are above } 58}$

4. (2 points) A certain electronic lock has ten buttons on its face, numbered from 0 to 9.

   (a) If you open the lock by pressing any three buttons one at a time in a definite sequence, how many different ways could you try to open the lock?

   order matters

   $\underline{10}\ \underline{10}\ \underline{10} = \boxed{1000}$

   with replacement (though I also accepted ${}_{10}P_3$)

   (b) If you open the lock by pressing three different buttons all at the same time, how many different ways could you try to open the lock?

   order doesn't matter

   ${}_{10}C_3 = \boxed{120}$

   without replacement

5. (3 points) Consider events A and B, where $P(A) = 0.2$, $P(B) = 0.6$, and $P(A|B) = 0.1$.

(a) Calculate $P(A$ or B).

$P(AB) = P(A|B)\,P(B)$

$= (0.1)(0.6) = 0.06$

$P(A \text{ or } B) = P(A) + P(B) - P(\text{both})$

$= 0.2 + 0.6 - 0.06 = \boxed{0.74}$

note: cannot say $P(AB) = P(A)P(B)$ since not independent

(b) Are events A and B independent? Explain briefly.

no, because $P(A) \neq P(A|B)$

(c) Are events A and B mutually exclusive? Explain briefly.

no, because $P(A|B) \neq 0$

6. (4 points) In a group of 10 batteries, 3 are dead. You choose 2 batteries at random.

(a) Which probability distribution would best describe the variable x, where x is the number of dead batteries you have chosen? Explain briefly.

hypergeometric because there are 2 trials and you are selecting without replacement

(b) Calculate the average number of dead batteries you will choose. Also, calculate the standard deviation for this number.

$\mu = n\left(\frac{M}{N}\right) = 2\left(\frac{3}{10}\right) = \boxed{0.6}$

$\sigma^2 = n\left(\frac{M}{N}\right)\left(\frac{N-m}{N}\right)\left(\frac{N-n}{N-1}\right)$

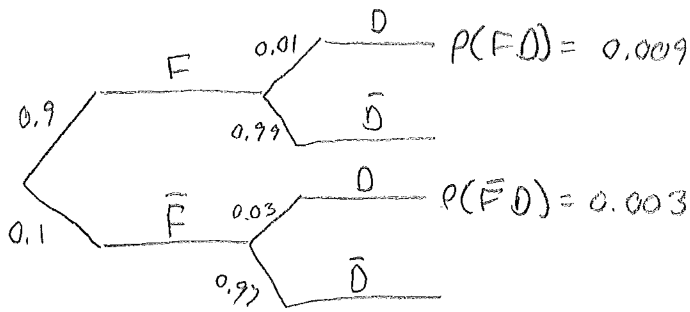
$= 2\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)\left(\frac{8}{9}\right) \approx 0.373$

so $\boxed{\sigma = 0.61}$

7. (7 points) A worker-operated machine produces a defective item one time out of one hundred if the worker follows the machines operating instructions exactly. However, if the worker does not follow instructions exactly, the probability of producing a defective item rises to three times out of one hundred. The worker follows the instructions 90% of the time.

   (a) What is the probability that the machine produces a defective item?

F = follows directions
D = defective



$$P(D) = P(FD) + P(\bar{F}D) = \boxed{0.012 \quad \text{or} \quad 1.2\%}$$

   (b) If a randomly selected item is defective, what is the probability that the worker didn't follow directions?

$$P(\bar{F} \mid D) = \frac{P(\bar{F}D)}{P(D)} = \frac{0.003}{0.012} = 0.25 \quad \text{or } 25\%$$

   (c) Are "producing a defective item" and "following directions" independent? Briefly explain your reasoning, including values of appropriate probabilities.

method #1:

$P(D) = 0.012$

$P(D \mid F) = 0.01$

method #2

$P(F) = 0.9$

$P(F \mid D) = \frac{P(FD)}{P(D)} = \frac{0.009}{0.012} = 0.75$

method #3

$P(FD) = 0.009$

$P(F)P(D) = (0.9)(0.012) = 0.0108$

since the two probabilities are unequal in each case, dependent

8. (4 points) Suppose that people's birthdays are evenly distributed throughout the days of the week. What is the probability that of any seven people, exactly one was born on a Monday? At least one?

binomial: 7 people, each with probability $\frac{1}{7}$ of having been born on a Monday

$$P(x=k) = {}_nC_k\, p^k q^{n-k}$$

$$P(x=1) = {}_7C_1 \left(\tfrac{1}{7}\right)^1 \left(\tfrac{6}{7}\right)^{7-1}$$

$$= \frac{46656}{117649} \approx 0.396569 \approx \boxed{40\%}$$

$$P(x \geq 1) = 1 - P(x=0)$$

$$= 1 - {}_7C_0 \left(\tfrac{1}{7}\right)^0 \left(\tfrac{6}{7}\right)^{7-0}$$

$$= \frac{543607}{823543} \approx 0.660083$$

$$\approx \boxed{66\%}$$

if detailed calculation is correct but there's a calculator error $-\frac{1}{2}$

if no ${}_7C_1$ -1