

STAT 254 – Test 2

February 24, 2017
Instructor: Patricia Wrean

Name: Solution Set

Total: 30 points

1. (3 points) A market researcher for Air Canada wishes to interview a random selection of passengers who flew with Air Canada in the past year. For the following situations, identify the survey method used to pick these passengers.

(a) The researcher randomly selects 10 flights from the past year, and interviews all passengers that were on each of those flights. cluster

(b) The researcher makes a list of all of the Air Canada passengers in the past year and randomly chooses 100 passengers from the list. simple random

(c) The researcher makes a list of all of the Air Canada passengers in the past year. The fifth passenger on the list is selected and then every 50th passenger after that.

1-in-50
systematic
random

2. (2 points) On an assembly line, it is known that 2% of the gizmos being made are defective. 150 gizmos are randomly selected and tested for defects. Let x be the number of defective gizmos. (2 points)

(a) What is the exact distribution of x ?

binomial

(b) Can you approximate this distribution with a normal distribution? Explain your reasoning.

$$np = 150(0.02) = 3$$

$$nq = 150 - 3 = 147$$

since $np < 5$, can't use normal.

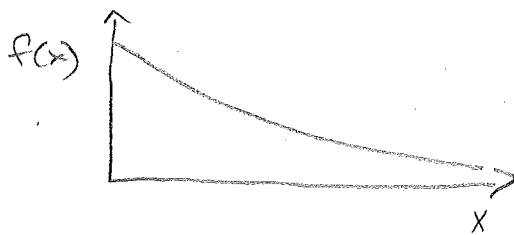
3. (5 points) The manager of a gas station has observed that the time required by a driver to fill their car's tank is exponentially distributed with a mean of 8 minutes.

(a) What is the probability that the time it takes a random driver to fill their tank is shorter than the mean time μ ? Give an exact answer.

$$f(x) = \begin{cases} k e^{-kx} & \text{for } x \geq 0 \\ 0 & \end{cases} \quad \text{where } k = \frac{1}{\mu} = \frac{1}{8}$$

$$\begin{aligned} P(x < \mu) &= \int_0^{\mu} f(x) dx = \int_0^{\mu} k e^{-kx} dx \\ &= -e^{-kx} \Big|_0^{\mu} \\ &= -e^{-k\mu} + e^0 \\ &= e^{-\frac{1}{8} \cdot 8} + 1 \\ &= 1 - e^{-1} \end{aligned}$$

(b) Why is your answer to part (a) not equal to 50%?



because the distribution isn't symmetrical (the mean is pulled towards the tail)

4. (2 points) On your way to this quiz, you stop at Tim Hortons and pick up a box of six doughnuts. Will the weight of the box be normally distributed if

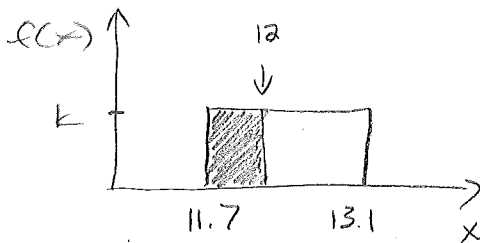
(a) the weight of each doughnut is normally distributed?

Yes No / Maybe

(b) the weight of each doughnut is skewed?

Yes No Maybe

5. (3 points) In a bottling plant, the volume of Pepsi in bottles filled by a certain machine is known to be uniformly distributed between 11.7 and 13.1 ounces. The label on the bottle says 12 ounces. What is the probability that a randomly selected bottle contains less Pepsi than the label claims?



$$\text{total area} = 1$$

$$k(13.1 - 11.7) = 1$$

$$k = \frac{1}{1.4}$$

$$P(x < 12) = \text{shaded area}$$

$$= k(12 - 11.7)$$

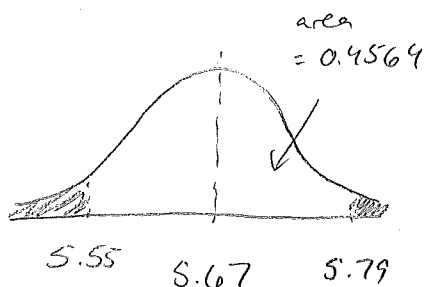
$$= \frac{1}{1.4}(0.3)$$

$$\approx 0.214286$$

21%

6. (6 points) Quarters have masses that are normally distributed with a mean of 5.67 g and a standard deviation of 0.070 g.

- (a) If a vending machine is adjusted to reject quarters whose mass is less than 5.55 g or more than 5.79 g, what is the probability that a legal quarter will be rejected?



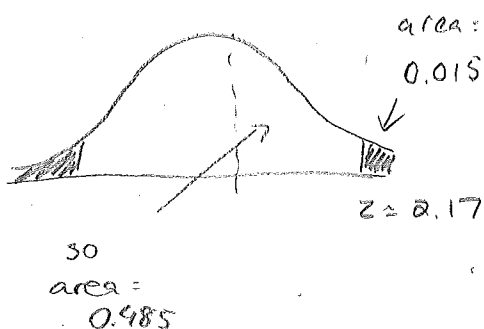
$$z_{\text{high}} = \frac{x - \mu}{\sigma} = \frac{5.79 - 5.67}{0.070} = 1.714$$

$$z_{\text{low}} = \frac{x - \mu}{\sigma} = \frac{5.55 - 5.67}{0.070} = -1.714$$

$$P(\text{rejected}) = 2(0.5 - 0.4564) = 0.0872$$

$$= 9.06$$

- (b) Find the range of accepted quarters if the machine is readjusted so that the lightest 1.5% and the heaviest 1.5% are rejected.



$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma$$

$$= 5.67 \pm (2.17)(0.070)$$

$$= 5.5181 \text{ to } 5.8219 \text{ g}$$

- (c) If you find that you have 100 quarters in your piggy bank at home, do you think that it's likely that the average mass of these quarters is more than 5.79 g? Explain your answer.



$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{5.79 - 5.67}{0.070/\sqrt{100}}$$

$$= 17.14$$

having a z-score this large is extremely unlikely

7. (4 points) Two groups of people were given a simple task to perform, one in silence, and the second with bad rap music blasting at them. The first group of 30 took 15 minutes on average (standard deviation 4 minutes) to complete the task, while the second group of 40 took 23 minutes on average (standard deviation 10 minutes) to complete it. Calculate a 99% confidence interval and comment on whether this interval gives sufficient evidence to indicate a difference in the average time to completion for the two groups.

$$\mu_2 - \mu_1 = \bar{x}_2 - \bar{x}_1 \pm Z_{\alpha/2} SE$$

$$\begin{aligned} \text{where } SE &= \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \\ &= \sqrt{\frac{4^2}{30} + \frac{10^2}{40}} \\ &= 1.74 \end{aligned}$$

$$\begin{aligned} \text{so } \mu_2 - \mu_1 &= 23 - 15 \pm 2.58 (1.74) \\ &= 8 \pm 4.5 \\ &= 8 \pm 5 \end{aligned}$$

The 99% confidence interval for the difference in the two groups is 3 to 13 minutes. Since the interval does not include zero, there is indeed a significant difference between the two groups.

8. (5 points) A researcher wishes to estimate the proportion of Canadians who are in favour of the Kyoto Protocol's call for the reduction of emissions of greenhouse gases. She wants to be 99% confident that her estimate is within 5% of the true proportion.

(a) How large a sample should be taken if in a prior survey of Canadians, 70% were in favour of the Kyoto protocol?

$$\text{MOE} \leq B$$

$$z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \leq B$$

$$z_{\alpha/2}^2 \frac{\hat{p}\hat{q}}{n} \leq B^2$$

$$n \geq \frac{\hat{p}\hat{q} z_{\alpha/2}^2}{B^2}$$

$$\geq \frac{(0.7)(0.3)(2.576)^2}{(0.05)^2}$$

$$\geq 557.905$$

$$\geq 558$$

(b) If no estimate of the sample proportion is available, how large should the sample be?

same calculation but $\hat{p} = 0.5$

$$n \geq \frac{\hat{p}\hat{q} z_{\alpha/2}^2}{B^2}$$

$$\geq \frac{(0.5)(0.5)(2.576)^2}{(0.05)^2}$$

$$\geq 643.578$$

$$\geq 664$$