STAT 254 – Test 2

March 29, 2018 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 25 points

1. (3 points) The manager for a large hotel chain wishes to survey guests who stayed at hotels in that chain within the last year. For the following situations, identify the sampling plan used to pick these guests.

   (a) The manager randomly selects 10 hotels and surveys all guests who stayed at those hotels within the last year.

   cluster

   (b) The manager makes a list of all of the guests who stayed at any of the chain's hotels within the past year and randomly chooses a certain number of guests from that list.

   simple random

   (c) For each hotel in the chain, the manager chooses a random selection of guests who stayed within the last year.

   stratified

2. (2 points) You are shopping for produce at your local grocery store.

   (a) At the supermarket, you pick up a package of blueberries, and note that there are about 100 blueberries per package. The produce clerk is a fan of statistics and tells you that the weights of the packages of blueberries are normally distributed. What does this tell you about the distribution of weights for the individual blueberries? (Circle one.)

   (i) They must be normally distributed.
   (ii) They must be skewed.
   (iii) They could have any distribution at all.

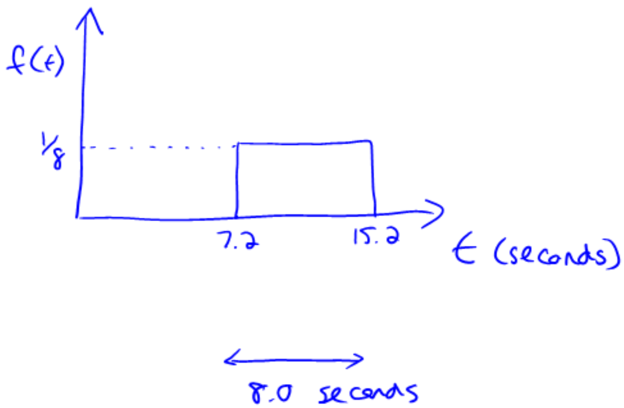
   since $n \geq 30$, any distribution of weights will have $\bar{x}$ normally distributed

   (b) Now you pick up a package of peaches, and each package contains exactly three peaches. The same clerk tells you that the weights of these packages are also normally distributed. What does that tell you about the distribution of weights for the individual peaches? Circle one.

   (i) They must be normally distributed.
   (ii) They must be skewed.
   (iii) They could have any distribution at all.

   since $n \ll 30$, peaches must be normally distributed to begin with for $\bar{x}$ to be normally distributed

3. (3 points) The boot time for a Microsoft Surface Pro 4 (I5) is known to be uniformly distributed between 7.2 and 15.2 seconds. Calculate the mean and standard deviation for the boot time for this device.



total area underneath curve is one, so height must be $\frac{1}{8.0}$

a) by symmetry, $\mu = \text{midpoint} = \frac{7.2+15.2}{2} = \boxed{11.2 \text{ seconds}}$

1

b) $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$= \int_{7.2}^{15.2} \frac{1}{8} x^2\,dx - 11.2^2$

$= \frac{1}{24} x^3 \Big|_{7.2}^{15.2} - 11.2^2$

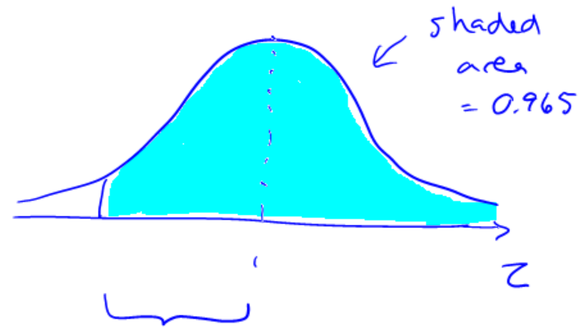
$= \frac{1}{24}\left(15.2^3 - 7.2^3\right) - 11.2^2$

$= 5.\overline{3}$

2

$\sigma = \sqrt{\sigma^2} \approx 2.3094 \approx \boxed{2.3 \text{ seconds}}$

4. (3 points) A single commercial jet engine uses an average of 718 gallons per hour of jet fuel once it is at its cruising altitude, with a standard deviation of 42 gallons per hour. When the jet is in cruising position, the fuel consumption follows a normal distribution. The fuel consumption will exceed a certain value 96.5% of the time. Find that value.



this area = 0.465

which has $z = -1.81$

$$z = \frac{x - \mu}{\sigma}$$

$$x = \mu + z\sigma$$

$$= 718 + (-1.81)(42)$$

$$= 641.98$$

≈ 642 gallons/hour

two-tailed

$z = -2.11$

$x = 629.38$

-1

positive z

$z = 1.81$

$x = 794.02$

-1

two-tailed and +z

$z = 2.11$

$x = 806.62$

-2

5. (3 points) A gambling casino records and plots the mean daily gain or loss from five blackjack tables on an $\overline{x}$ chart. The overall mean of the sample means and the standard deviation of the combined data over 40 weeks were $\overline{\overline{x}} = \$10{,}752$ and $s = \$1605$, respectively.

   (a) Calculate the UCL and LCL for this chart.

$$UCL = \overline{\overline{x}} + 3\,SE$$

$$= \overline{\overline{x}} + 3\frac{s}{\sqrt{n}}$$

$$= 10752 + \frac{3(1605)}{\sqrt{5}}$$

$$= 10752 + 2153.32 = \boxed{\$12905.33}$$

2

$$LCL = \overline{\overline{x}} - 3\,SE$$

$$= \boxed{\$8598.67}$$

   (b) Five readings in a row were between \$9000 and \$9500. Should the manager of the casino be concerned? Explain briefly.

No, because \$9000 - \$9500 is above the LCL.

6. (5 points) A dentist is interested in finding out what fraction of appointments in her practice are cancelled. Her records for the past month indicate that 15 out of the 132 appointments were cancelled.

(a) Calculate a 90% confidence level for the proportion of cancellations for this dentist's appointments.

$$p = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \quad 1$$

$$= \frac{15}{132} \pm 1.645\sqrt{\left(\frac{15}{132}\right)\left(\frac{117}{132}\right)\left(\frac{1}{132}\right)}$$

$$= 0.113636 \pm 0.045441$$

$$= 0.068196 \text{ to } 0.159077$$

check: 1

$np = 15$
$nq = 132-15$ $\Big\} > 5$ ✓

$z_{\alpha/2} = 1.645$ for 90%

1

90% CI: $\boxed{\begin{array}{l} 6.8\% \text{ to } 15.9\% \\ \text{or } 7\% \text{ to } 16\% \end{array}}$ 1

(b) A trade publication claims that 15% of all dental appointments are cancelled. Is this consistent with the data collected by the dentist? Explain briefly.

Yes, because 15% is contained within the confidence interval. 1

7. (6 points) A large shipping company has a fleet of seventy identical cargo ships. A fluid dynamics engineer claims that a certain modification of the front hull will increase a ship's speed for the same engine settings. The fleet is randomly divided into two groups of equal size, and the first group is modified while the second group is left unchanged. When the engines are at a certain setting, the mean speed in a calm sea for the unmodified ships is 25.6 knots with standard deviation 4.3 knots, while the modified ships have a mean speed of 28.7 knots with standard deviation 5.0 knots.

(a) Is the engineer correct that the modifications have increased the ship speed? Use a p-test.

H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

H_a: $\mu_1 - \mu > 0$ $\quad \mu_1 > \mu_2$ 1

(−1) no hypotheses

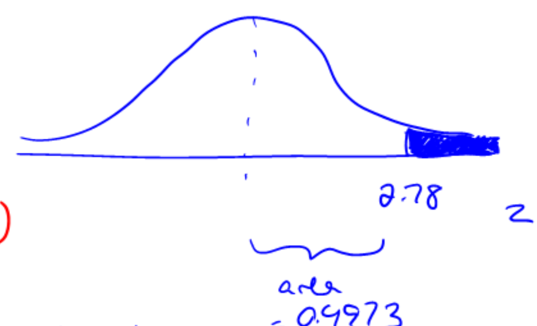
(−½) $\bar{x}$ instead of μ

(−1) p instead of μ

test statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ 1

$$= \frac{28.7 - 25.6}{\sqrt{\frac{5.0^2}{35} + \frac{4.3^2}{35}}} = 2.781$$ 1



$p = 0.5 - 0.4973 = 0.0027$ 1

"highly significant"

The engineer is correct – the increase in speed is highly significant 1

(b) If the results of this hypothesis test later turn out to be incorrect, what type of error was made?

Type I : the test rejected H_0 when H_0 was actually true 1