Math 254
Practice Questions for Test 1

1. Find the sample mean $\bar{x}$, and standard deviation s for the following sample data set.

$$
\begin{array}{cccccccccc}
1.8 & 1.5 & 1.4 & 1.6 & 1.1 & 1.6 & 1.8 & 1.7 & 1.3 & 1.5 \\
1.3 & 1.2 & 1.8 & 1.4 & 2.1 & 1.5 & 1.3 & 1.4 & 1.5 & 1.0
\end{array}
$$

2. Construct a box-plot for this data set.

$$
3 \quad 15 \quad 18 \quad 19 \quad 19 \quad 20 \quad 21 \quad 22 \quad 26 \quad 27 \quad 29
$$

3. A set of data has a mean of 60 and a standard deviation of 5. You know nothing else about the size of the data or the shape of the distribution.

   (a) What can you say about the proportion of measurements that fall between 45 and 75?

   (b) What can you say about the proportion of measurements that are above 70?

4. From a group of 6 men and 8 women, how many different 6-person committees consisting of 3 men and 3 women can be formed if:

   (a) There are no other restrictions?

   (b) Two of the women are feuding and refuse to serve on the committee together?

   Note: A committee is an unordered group.

5. According to the weather service, there is a 50% chance of rain in New York and a 30% chance of rain in Tokyo. Assuming that New York's weather is independent of Tokyo's weather, find the probability that it will rain in at least one of these cities.

6. Let A and B be two events such that

$$
P(A) = 0.4, \; P(B) = 0.5, \; \text{and } P(A \cup B) = 0.8.
$$

   (a) Compute $P(A \cap B)$.

   (b) Are A and B independent?

   (c) Compute $P(B \mid A^c)$.

7. A fair die is rolled 10 times. Find the probability that the number 6 will appear

   (a) exactly three times;

   (b) at least twice.

8. Which event is more likely: rolling at least one six in four throws of one die, or rolling at least one double-six in 24 throws of a pair of dice?

9. A multiple choice test consists of 10 questions with 5 choices for each question. What is the probability of getting exactly 6 questions right by randomly guessing all of them?

10. Three out of every four email in my mailbox are spam (junk emails). My internet service provider has a spam-detector which adds the keyword "SPAM" in the title so that junk emails can be filtered and deleted easily. However, the spam-detector is only 95% accurate. If an email is tagged as spam, what is the probability that it actually is spam?

11. Box I contains 3 white and 4 red balls, whereas box II contains 1 white and 2 red balls. A ball is randomly chosen from box I and put into box II, and a ball is then randomly selected from box II.

    (a) What is the probability that the ball selected from box II is white?

    (b) What is the conditional probability that the transferred ball was white, given that a white ball was selected from box II?

12. A communications channel transmits messages as bit-strings. Due to random noise, each bit transmitted is incorrectly received with probability 10%. To reduce the chance of error, we send 00000 instead of 0 and 11111 instead of 1, and the receiver of a message uses "majority" decoding (e.g., 10110 is decoded as 1). If the message 110 is sent as 11111 11111 00000, what is the probability that it will be correctly decoded by the receiver?

13. A certain hockey player scores on average 0.548 goals per game. Suppose the number of goals scored in a game has a Poisson distribution.

    (a) Find the probability that the player will score 3 or more goals during a certain game.

    (b) If the player plays 82 games in a season, what's the probability that at least one 3 goals or more game occurs?

14. Suppose 300 typos are distributed randomly throughout a book of 500 pages. We can assume that the number of typos in a given page of the book has a Poisson distribution. Find the probability that a given page contains:
    (a) exactly 2 typos,   (b) 2 or more typos.

15. Each of 12 refrigerators of a certain type has been returned to a distributor because of the presence of a high-pitched oscillating noise when the refrigerator is running. Suppose that 5 of these 12 have defective compressors and the other 7 have less serious problems. If they are examined in random order, let X be the number among the first 6 examined that have a defective compressor. Compute the following.
    (a) $P(X = 1)$ (b) $P(1 \leq X \leq 3)$

16. Site A and site B are two potential sites for an oil company to drill a well. The cost of drilling a well at site A is \$4 million, while at site B the cost is \$6 million. The well at site A will produce \$30 million in revenue with probability 0.3, \$10 million in revenue with probability 0.6, and no revenue with probability 0.1. The well at site B will produce \$22 million in revenue with probability 0.8, and no revenue with probability 0.2. Based on expected revenue, which site should the oil company choose?