# Math 254
## Practice Questions for Test 2

1. A coffee machine is regulated to fill an average of $250\,\mathrm{ml}$ per cup. The amount of coffee per cup is normally distributed with a standard deviation of $12\,\mathrm{ml}$.

   (a) What percentage of cups will contain less than 238 ml?

   (b) Find $v$ such that only 5% of the cups have a volume larger than $v$.

   (c) Find the probability that a cup of coffee contains between $248\,\mathrm{ml}$ and $252\,\mathrm{ml}$.

   (d) If a random sample of 36 cups of coffee is taken, find the probability that the sample mean is between $248\,\mathrm{ml}$ and $252\,\mathrm{ml}$.

   (e) If a random sample 10 cups of coffee is taken, find the probability that the total volume of the sample is between 2480 ml and 2520 ml.

   *Answers:* (a) 15.9%   (b) $v = 269.7\,\mathrm{ml}$   (c) 13.2%   (d) 68.3%   (e) 40.2%

2. A pair of fair dice is rolled 240 times. Use a normal approximation to estimate the probability that a sum of 7 occurs   (a) at least 35 times;   (b) between 32 and 45 times inclusive.
   *Answers:* (a) 83%   (b) 76%

3. Suppose the records of a hotel show that, on average, 10% of their prospective guests will not claim their reservations. If the hotel accepts 215 reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?
   *Answer:* 94.4%

4. Fifty pipes were stressed for failures and we found a failure strength sample mean of $\overline{x} = 5152$ psi. It is known from long experience with similar pipes that $\sigma = 250$ psi.

   (a) Find a 95% confidence interval for the mean failure strength.

   (b) We need to find a 95% confidence interval with a width no greater than 100 psi. What sample size should we use?

   *Answers:* (a) $(5152 \pm 69.3)\,\mathrm{psi} \implies (5082.7, 5221.3)$   (b) $n \geq 97$

5. A car dealer has found the the length of time before a major repair is required on the cars it sells is normally distributed, with a mean of 36 months and standard deviation of 9 months. If the dealer wants only 5% of the cars to fail before the end of the guarantee period, for how many months should the cars be guaranteed?
   *Answer:* 21 months

6. A random sample of 20 observations from a normal population is the following.

   $$13.2 \quad 9.1 \quad 7.5 \quad 11.5 \quad 9.8 \quad 10.4 \quad 12.5 \quad 9.7 \quad 10.4 \quad 11.1$$
   $$8.9 \quad 10.5 \quad 10.1 \quad 12.8 \quad 8.6 \quad 9.2 \quad 13.8 \quad 10.5 \quad 9.6 \quad 10.8$$

   (a) Find a 95% confidence interval for the population mean $\mu$.

   (b) Find a 95% confidence interval for the population standard deviation $\sigma$.

   (c) Test $H_0\colon \mu = 10$ vs $H_a\colon \mu > 10$, using $\alpha = 5\%$. Estimate the $p$-value of the test?

   *Answers:* (a) $10.5 \pm 0.76 \implies (9.74, 11.26)$   (b) $(1.23, 2.37)$   (c) Don't reject $H_0$, $5\% < p\text{-value} < 10\%$

7. A coin is tossed 50 times, resulting in 15 heads. Is this sufficient evidence to reject the assumption that the coin is fair? Give a $p$-value to support your argument.
   *Answer:* We test $H_0\colon p = 0.5$ vs $H_a\colon p \neq 0.5$ and we obtain a $p$-value of 0.0046. We reject $H_0$.

8. An experiment was made to compare two population means. The experiment results are the following.

   | Population | $n$ | $\overline{x}$ | $s$ |
   |---|---|---|---|
   | 1 | 45 | 38.4 | 5.1 |
   | 2 | 42 | 36.1 | 4.8 |

(a) Test $H_0\colon \mu_1 = \mu_2$ vs $H_a\colon \mu_1 > \mu_2$, at the 5% level. Give the $p$-value of the test.

(b) Find a 95% confidence interval for $\mu_1 - \mu_2$.

*Answers:* (a) Reject $H_0$, $p$-value $= 1.5\%$ (b) $2.3 \pm 2.08 \implies (0.22, 4.38)$.

9. To test the effectiveness of taking aspirin daily in reducing heart attacks, two groups were considered over a period of time. The following results were obtained.

| Group | Sample size | Heart attack | No heart attack |
|---|---|---|---|
| Placebo | 11,034 | 239 | 10,795 |
| Aspirin | 11,037 | 139 | 10,898 |

Conduct an appropriate test of hypothesis and state your conclusion by supporting it with a $p$-value.

*Answer:* We test $H_0\colon p_1 = p_2$ vs $H_a\colon p_1 > p_2$ and we obtain: $p$-value $= P(Z > 5.19) = 1.05 \times 10^{-7}$. This is highly significant and we reject $H_0$.

10. The average life of a certain type and brand of battery is claimed to be 75 weeks. The average life of each of 9 randomly selected batteries is listed below. Assume the battery life distribution is normal. It is of interest to know if the sample data suggest the average life is smaller than 75 weeks.

$$74.5, \ 75.0, \ 72.3, \ 76.0, \ 75.2, \ 75.1, \ 75.3, \ 74.9, \ 74.8$$

(a) State the appropriate hypotheses.

(b) Test them at the 5% level and estimate the $p$-value.

*Answers:* (a) We test $H_0\colon \mu = 75$ vs $H_a\colon \mu < 75$    (b) We don't reject $H_0$, $p$-value $> 10\%$

11. The mean driving times (in minutes) on 5 different days for Alice and Bob to drive to their classes at Camosun college are listed below. Is there sufficient evidence to conclude that the mean driving time for both students differ? Assume the population distributions are normal with equal variances. Use a 0.01 significant level.

| Alice | 18.9 | 13.8 | 18.6 | 22.5 | 23.1 |
|---|---|---|---|---|---|
| Bob | 22.8 | 23.1 | 22.7 | 21.9 | 26.7 |

*Answer:* We test $H_0\colon \mu_1 - \mu_2 = 0$ vs $H_a\colon \mu_1 - \mu_2 \neq 0$. We do not reject $H_0$, there is not enough evidence to conclude that the driving times for Alice and Bob differ.

12. A national survey states that 67% of college students are under the age of 25, 21% are between 25 and 30, 8% are between 30 and 40, and 4% are over 40. A random sample of 250 Camosun College students yielded the following data:

| Age | under 25 | 25 − 30 | 30 − 40 | over 40 |
|---|---|---|---|---|
| Frequency | 138 | 62 | 32 | 18 |

(a) State the appropriate hypotheses to test whether the distribution of students at Camosun agrees with the national survey.

(b) Test them at the 5% level and state the conclusion.

*Answers:* (a) $H_0\colon p_1 = 0.67, \ p_2 = 0.21, \ p_3 = 0.08, \ p_4 = 0.04$ vs $H_a\colon$ at least one $p_i$ is different from the specified values.    (b) We reject $H_0$ and conclude that the distribution of students' age at Camosun does not agree with the national survey.

13. A random sample from a normal population produced the following measurements.

$$2.1 \quad 4.3 \quad 2.4 \quad 2.7 \quad 4.0 \quad 3.5 \quad 3.6$$

Test $H_0\colon \sigma^2 = 0.8$ versus $H_a\colon \sigma^2 \neq 0.8$ at the 5% significance level. Estimate the $p$-value.

*Answer:* We obtain $\chi^2_{\text{obs}} = 5.24$. We can estimate that $p$-value $> 20\%$. Therefore, we don't reject $H_0$.