STAT 254 – Makeup Test 1

February 15, 2019 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 25 points

1. (3 points) A statistician working for BC Ferries wants to study the time it takes to complete the ferry sailing from Vancouver to Victoria. To do so, she randomly selects fifty ferry sailings on the route from Vancouver to Victoria within the last year and records the time it took the ferry to complete the trip.

   (a) This data is (circle one)

   (i) univariate
   (ii) bivariate
   (iii) multivariate

   (b) Consider the following list:

   (i) one ferry sailing on that route during that year
   (ii) the time for one ferry sailing on that route during that year
   (iii) the times for the fifty ferry sailings selected on that route during that year
   (iv) the times for all ferry sailings on that route during that year
   (v) the times for all ferry sailings on that route
   (vi) the times for all ferry sailings in BC

   Which entry from the above list is the sample? (iii)

   Which entry from the above list is the experimental unit? (i)

   if ii) (-½)
   if anything else, (-1)

2. (3 points) You have a set of sample data which consists of temperatures in degrees Celsius. The mean is 18.0°C with a standard deviation of 4.5~~°F~~ °C. Each temperature is converted to degrees Fahrenheit by first multiplying by $\frac{9}{5}$ and then adding 32. What are the new mean and standard deviation of the data set after the conversion?

$$\text{new mean} = \frac{9}{5}(\text{old mean}) + 32 = \frac{9}{5}(18.0) + 32 = 64.4\,°F$$

$$\text{new SD} : \frac{9}{5}(\text{old SD}) = \frac{9}{5}(4.5) = 8.1\,°C$$

shift in data set doesn't change SD

mean: 64.4°F

standard deviation: 8.1 °F

3. (3 points) Suppose that $P(A) = 0.4$ and $P(B) = 0.5$. For each situation below, state whether A and B are independent or dependent. Explain briefly.

   (a) $P(AB) = 0.4$

method #1:

$P(A)P(B) = 0.2$

$P(AB) = 0.4$

not equal

$\therefore$ $\boxed{\text{dependent}}$

method #2:

$P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.4}{0.5} = 0.8$

$P(A) = 0.4$

not equal $\therefore$ $\boxed{\text{dependent}}$

method #3:

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.4}{0.4} = 1$

$P(B) = 0.5$

not equal $\therefore$ $\boxed{\text{dependent}}$

   (b) $P(A \text{ or } B) = 0.7$

$P(A \text{ or } B) = P(A) + P(B) - P(AB)$

$0.7 = 0.4 + 0.5 - P(AB)$

$P(AB) = 0.2$

$\boxed{\text{independent}}$

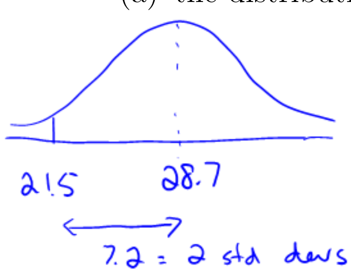
and use same methodology as above to show that one of

$P(AB) = P(A)P(B)$ or $P(A|B) = P(A)$ or $P(B|A) = P(B)$

$0.2 = (0.4)(0.5)$ $\quad \frac{0.2}{0.5} = 0.4$ $\quad \frac{0.2}{0.4} = 0.5$

4. (2 points) The time it takes to drive from the Lansdowne campus to the Interurban campus during the day has a mean of 28.7 minutes and a standard deviation of 3.6 minutes. If you were to make this drive repeatedly, what can you say about the number of trips that would take less than 21.5 minutes if

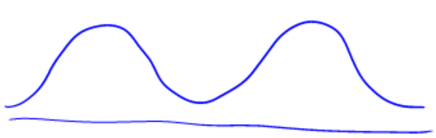
   (a) the distribution of times is unimodal and symmetrical?



Empirical: ~95% within 2 std devs

so $\boxed{\sim 2.5\% \text{ below}}$

   (b) the distribution of times is bimodal and symmetrical?



Tchebysheff: $\geq 75\%$ within 2 std devs

so $\boxed{\leq 12.5\% \text{ below}}$

due to symmetry

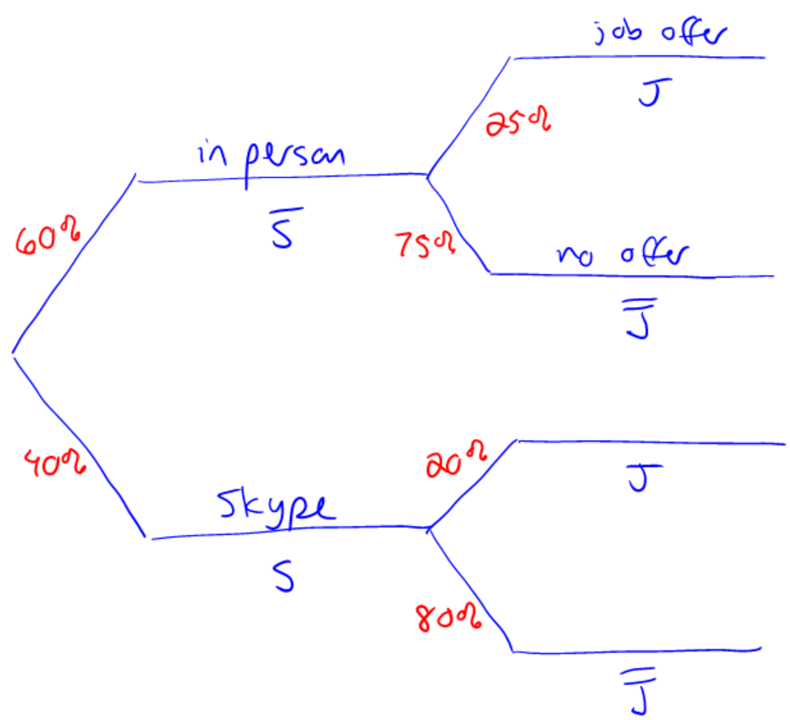
(−½) if didn't include ~ or ≤

(−½) no symmetry

5. (4 points) A large engineering firm conducts 60% of its job interviews in person and the rest via Skype. 25% of the in-person interviews result in a job offer, while 20% of the Skype interviews do.

   (a) For a randomly selected interview, what's the probability that it was done over Skype or results in a job offer or both?

   (b) Suppose that a randomly selected interview resulted in a job offer. What's the probability that the interview was done via Skype?



$P(\bar{S}J) = (0.6)(0.25) = 0.15$

$P(\bar{S}\bar{J}) = (0.6)(0.25) = 0.45$

$P(SJ) = (0.4)(0.2) = 0.08$

$P(S\bar{J}) = (0.4)(0.8) = 0.32$

a) $P(S \text{ or } J) = P(SJ) + P(S\bar{J}) + P(\bar{S}J)$

$= 0.08 + 0.32 + 0.15$

$= 0.55$

b) $P(S \mid J) = \dfrac{P(SJ)}{P(J)} = \dfrac{P(SJ)}{P(SJ) + P(\bar{S}J)}$

$= \dfrac{0.08}{0.08 + 0.15}$

$= 0.347826$ or $\boxed{35\%}$

6. (2 points) Consider a typical episode of the Mythbusters TV series, which is famous for the number of ways that they blow things up. What probability distribution best describes x, the number of explosions per episode? Explain briefly.

Poisson - number of events in a given time

or no maximum number of explosions (unbounded)

7. (4 points) You are designing a raffle for your favourite charity. You are planning to sell 1000 tickets in total, with the grand prize being a Kindle valued at \$200 and six runner-up prizes of \$50 movie ticket vouchers. If you want your average profit per ticket to be fifty cents, how much should you sell each ticket for?

x	$p(x)$
$y-200$	$1/1000$
$y-50$	$6/1000$
y	$993/1000$

let y = ticket price

$E(x) = \sum x\, p(x)$

$0.5 = (y-200)\cdot\frac{1}{1000} + (y-50)\frac{6}{1000} + y\cdot\frac{993}{1000}$

$0.5 = y - 0.2 - 0.3$

so $y = 1$

Price of each ticket should be \$1.

8. (4 points) In a recent psychological study, individuals are presented with three different glasses of soft drink, labeled A, B, and C. They are asked to taste all three and then list them in order of preference. Suppose that the same soft drink has actually been put into all three glasses.

   Are "putting A first on the list" and "putting C last on the list" independent? Explain your reasoning, including values of appropriate probabilities.

sample space : ABC BAC CAB
ACB BCA CBA

all equally probable

$P(\text{A first}) = \frac{n(\text{A first})}{n_{tot}} = \frac{2}{6} = \frac{1}{3}$

$P(\text{A first} \mid \text{C last}) = \frac{n(\text{A first } \underline{\text{and}} \text{ C last})}{n(\text{C last})} = \frac{1}{2}$

since $P(\text{A first}) \neq P(\text{A first} \mid \text{C last})$

these events are <u>dependent</u>

<u>or</u> $P(\text{A first}) = \frac{1}{3}$ as above

$P(\text{C last}) = 1/3$ also

$P(\text{A first } \underline{\text{and}} \text{ C last}) = \frac{n(\text{both})}{n_{tot}} = \frac{1}{6}$

$P(\text{both}) \stackrel{?}{=} P(\text{A first}) \cdot P(\text{C last})$

$\frac{1}{6} \stackrel{?}{=} 1/3 \cdot 1/3$

no, so events are <u>dependent</u>