STAT 254 – Test 2

March ~~8~~ 21, 2019 **Name:** Solution Set

Instructor: Patricia Wrean

Total: 25 points

1. (3 points) A medical researcher wishes to study the patient outcomes for certain operations at BC hospitals within the last year. For the following situations, identify the survey method used to pick these operations.

   (a) For each hospital in BC, the researcher chooses a random selection of operations that occurred at that hospital within the last year.

   stratified

   (b) The researcher randomly selects 10 hospitals in BC and studies all operations that occurred at those hospitals within the last year.

   cluster

   (c) The researcher makes a list of all of the operations that took place in BC within the past year and randomly chooses a certain number of operations from that list.

   simple random

2. (2 points) Thirty percent of all calls coming into a telephone exchange are long-distance calls. Consider the next 200 calls that come into the exchange. Let x be the number of long-distance calls.

   (a) What is the name of the exact distribution of x?

   binomial (200 identical trials, prob of success same from trial to trial) 1

   (b) Can you approximate this distribution with a normal distribution? Explain briefly.

   $$\left.\begin{aligned} np &= 200(0.3) = 60 \\ nq &= 200(0.7) = 140 \end{aligned}\right\} > 5 \checkmark$$

   1

   yes

   if said this <u>and</u> $n \geq 30$ $-\frac{1}{2}$

3. (5 points) In a certain city, the time it takes in hours to repair a small hole in the sidewalk is a continuous random variable with probability density function

$$f(x) = \begin{cases} k(x^4 - x^5) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k such that $f(x)$ is indeed a probability density function.

$1 = \int_{-\infty}^{\infty} f(x)\,dx$

$1 = \int_0^1 k(x^4 - x^5)\,dx$

$1 = k\frac{x^5}{5} - k\frac{x^6}{6}$

$1 = k\left(\frac{1}{5} - \frac{1}{6}\right)$

$1 = \frac{k}{30}$

$\boxed{k = 30}$

2

(b) Find the probability that it takes at least half an hour to repair a hole in the sidewalk.

$P(x > \tfrac{1}{2}) = \int_{1/2}^{\infty} f(x)\,dx$

$= \int_{1/2}^{1} 30(x^4 - x^5)\,dx$

$= (6x^4 - 5x^5)\Big|_{1/2}^{1}$

$= (6-5) - \left(\frac{6}{16} - \frac{5}{32}\right)$

$P(x > \tfrac{1}{2}) = 57/64$

$= 0.890625$

$\boxed{\approx 89\%}$

2

"at most"

-1

(c) On average, how long does it take to repair a hole in the sidewalk?

$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^1 x\,30(x^4 - x^5)\,dx$

$= 30\left(\frac{x^6}{6} - \frac{x^7}{7}\right)\Big|_0^1$

$= 30\left(\frac{1}{6} - \frac{1}{7}\right)$

$\mu = \frac{30}{42} = 5/7$

≈ 0.71426

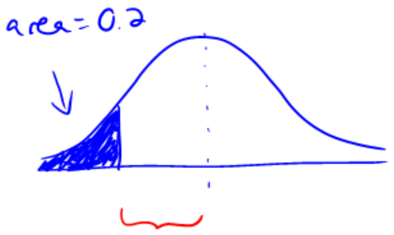
$\boxed{\approx 0.71 \text{ hours}}$

(just under 43 minutes)

1

4. (5 points) The mayor of Victoria was informed that household water usage is a normally distributed random variable with mean of 25 gallons/day and a standard deviation of 6 gallons/day.

   (a) If the mayor wants to give a tax rebate to the lowest 20% of water users, what should the gallons/day cutoff be?



area = 0.3
with z = −0.84
from normal table

$$z = \frac{x-\mu}{\sigma}$$

$$x = \mu + z\sigma = 25 + (-0.84)(6) = 19.96$$

$$\boxed{\approx 20 \text{ gallons/day}}$$

2

   (b) Calculate the probability that one hundred randomly-chosen households will use on average more than 27 gallons per day.

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{27-25}{6/\sqrt{100}} \approx 3.33$$

$$P(z > 3.33) = 0.5 - 0.4996 = 0.0004$$

$$\boxed{0.04\%}$$

2

   (c) As a civil engineer, you are brought in to consult on the system supplying Victorias households with water. Do you need to worry about whether on any particular day all 100,000 households will suddenly need more than 27 gallons per day each? Explain your answer.

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{27-25}{6/\sqrt{100000}} \approx 105$$ which is extremely unlikely!

1

so $\boxed{\text{no}}$

answer as if Q asks n = 100 (−½)

too vague (−1)

5. (5 points) A study conducted by the doctors of a particular hospital involved monitoring a random sample of 15 patients. The results showed the average amount of anesthetic needed to put a patient to sleep before surgery was 3.2 cc with a standard deviation of 0.4 cc. Previous research indicates that the amount of anesthetic needed is normally distributed.

   (a) State the conditions under which you are computing the confidence intervals in parts (b) and (c).

   (b) Give a 99% confidence interval for the average amount of anesthetic needed.

   (c) Give a 99% confidence interval for the standard deviation of the amount of anesthetic needed.

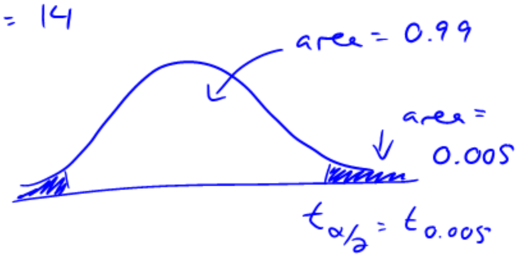
a) pop is normal

σ unknown but s is known

1

b) $\mu = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

df = 14



$= 3.2 \pm \frac{2.977(0.4)}{\sqrt{15}}$

$= 3.2 \pm 0.307$ cc

2

$\boxed{\text{CI: } \mu \text{ is from } 2.9 \text{ to } 3.5 \text{ cc}}$

c) $\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$

df = 14

$\frac{14(0.4)^2}{31.319} < \sigma^2 < \frac{14(0.4)^2}{4.075}$

2

$0.267 < \sigma < 0.741413$

$\boxed{\text{CI: } \sigma \text{ is between } 0.3 \text{ and } 0.7 \text{ cc}}$

no units $-\frac{1}{2}$

not written as interval $-\frac{1}{2}$

6. (2 points) On your way to this quiz, you stop at Tim Hortons and pick up a box of six doughnuts. Is it reasonable to assume that the weight of the box be normally distributed if

   (a) the weight of each doughnut is normally distributed? Yes / No

   (b) the weight of each doughnut is skewed? Yes / No

7. (3 points) A preliminary survey indicates that 25% of all airline flights arrive late. However, airline executives wish to do a bigger survey to determine the proportion of late flights to within ±2% with 90% confidence. How many flights must this bigger survey include?

$$p = \hat{p} \pm \underbrace{z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}}_{\text{want this } \le 2\%}$$

$$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \le B$$

$$\frac{z_{\alpha/2}}{B}\sqrt{\hat{p}\hat{q}} \le \sqrt{n}$$

$$n \ge \left(\frac{z_{\alpha/2}}{B}\right)^2 \hat{p}\hat{q}$$

$$\ge \left(\frac{1.645}{0.02}\right)^2 (0.25)(0.75)$$

$$\ge 1268.45$$

$$\ge 1269 \text{ flights}$$

(I'd say 1270 or maybe even 1300 myself.)

if say

$$z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le B \quad -2$$

round down -1

$n < 1269$ $-\frac{1}{2}$