

## Math 252: Formula Sheet for Tests and Final Exam

$$\int \tan x \, dx = \ln |\sec x| + C \qquad \int \cot x \, dx = -\ln |\csc x| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C \qquad \int \csc x \, dx = -\ln |\csc x + \cot x| + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C \qquad \int \csc x \cot x \, dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$


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$$\sin^2 \theta + \cos^2 \theta = 1 \qquad 1 + \tan^2 \theta = \sec^2 \theta \qquad 1 + \cot^2 \theta = \csc^2 \theta$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \qquad \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \qquad \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$


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$$e^{\int P(x) \, dx} \qquad y = vx \qquad u = y^{1-n} \qquad e^{\int \frac{My - Nx}{N} \, dx} \qquad e^{\int \frac{Nx - My}{M} \, dy}$$


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$$y_2 = y_1 \int \frac{e^{-\int P(x) \, dx}}{y_1^2} \, dx$$


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$$y = e^{\alpha x} (c_1 \cos(\beta x) + c_2 \sin(\beta x))$$

$$y = x^\alpha (c_1 \cos(\beta \ln x) + c_2 \sin(\beta \ln x))$$


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$$y_p = u_1 y_1 + u_2 y_2, \quad u_1 = \int \frac{W_1}{W} \, dx, \quad W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$$


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$$mx''(t) + \beta x'(t) + kx(t) = F_e(t)$$


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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots, \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots, \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$


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$$\mathbf{X}(t) = c_1 \mathbf{K}_1 e^{\lambda_1 t} + c_2 \mathbf{K}_2 e^{\lambda_2 t}$$

$$\mathbf{X}(t) = c_1 \mathbf{K} e^{\lambda t} + c_2 (\mathbf{K}t + \mathbf{P}) e^{\lambda t}$$

$$\mathbf{X}(t) = c_1 e^{\alpha t} [\mathbf{B}_1 \cos(\beta t) - \mathbf{B}_2 \sin(\beta t)] + c_2 e^{\alpha t} [\mathbf{B}_2 \cos(\beta t) + \mathbf{B}_1 \sin(\beta t)]$$


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$$\mathbf{X}_p(t) = \Phi(t) \int \Phi^{-1}(t) \mathbf{F}(t) \, dt$$


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## Table of Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$

| $f(t)$   | $\mathcal{L}\{f(t)\} = F(s)$           | $f(t)$                  | $\mathcal{L}\{f(t)\} = F(s)$                |
|--|--|-------------------------|---|
| 1  | $\frac{1}{s}$                          | $e^{at}$                | $\frac{1}{s-a}$                             |
| $t^n$  | $\frac{n!}{s^{n+1}}$                   | $\frac{e^{at} t^n}{n!}$ | $\frac{1}{(s-a)^{n+1}}$                     |
| $\sin \omega t$  | $\frac{\omega}{s^2 + \omega^2}$        | $\cos \omega t$         | $\frac{s}{s^2 + \omega^2}$                  |
| $e^{at} \sin \omega t$                                     | $\frac{\omega}{(s-a)^2 + \omega^2}$    | $e^{at} \cos \omega t$  | $\frac{s-a}{(s-a)^2 + \omega^2}$            |
| $t \sin \omega t$  | $\frac{2\omega s}{(s^2 + \omega^2)^2}$ | $t \cos \omega t$       | $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ |
| $\frac{\sin \omega t - \omega t \cos \omega t}{2\omega^3}$ | $\frac{1}{(s^2 + \omega^2)^2}$         | $\mathcal{U}(t-a)$      | $\frac{e^{-as}}{s}$                         |
| $\delta(t)$  | 1                                      | $\delta(t-a)$           | $e^{-as}$                                   |

$$\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \alpha \mathcal{L}\{f(t)\} + \beta \mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a) \quad \mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathcal{L}^{-1}\{e^{-as} F(s)\} = f(t-a) \mathcal{U}(t-a) \quad \mathcal{L}\{f(t) \mathcal{U}(t-a)\} = e^{-as} \mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) \quad \mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$$

$$f(t) * g(t) = \int_0^t f(\theta) g(t-\theta) d\theta \implies \mathcal{L}\{f(t) * g(t)\} = F(s)G(s)$$

$$\mathcal{L}\left\{\int_0^t f(\theta) d\theta\right\} = \frac{F(s)}{s}$$

$$f(t) \text{ has period } T \implies \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt$$