

Term: Winter, 2021

Name: \_\_\_\_\_

Instructor: Patricia Wrean

## Math 252-DX01

### Test 3

Total =  $\overline{20}$

- **Show your work.** All of the work on this test must be your own. While writing this test, you may not consult any other person, website, or other resource not listed below. If you have a question during the test, you may email me.
- Here is a list of the resources that you are allowed to use during this test:
  - your own notes
  - lecture notes, videos, handouts, practice questions, and practice tests from either my website at <http://wrean.ca/math252> or the Math 252 websites of any of the other instructors linked on the landing page of my site
  - your textbook (Zill), or any of the texts listed on the Textbook page at [http://wrean.ca/math252/math252\\_textbook.htm](http://wrean.ca/math252/math252_textbook.htm)
  - the Math 252 D2L website
  - the Math 252 WeBWorK online homework site
  - a scientific calculator. You may not use a calculator with graphing capability. If you like, you may use a scientific calculator app like the one at Desmos: <https://www.desmos.com/scientific>
  - a handy reference is the Math 252 Formula Sheet at [http://wrean.ca/math252/tests/math252\\_formula.pdf](http://wrean.ca/math252/tests/math252_formula.pdf)
  - if you have questions during the test, you may email me
- To submit this test, please use the Dropbox feature in the Assignments tab of D2L. Please assemble your answers into a single PDF or Word document, unless you've made other arrangements with me beforehand. Helpful software:
  - Genius Scan app at <https://www.thegrizzlylabs.com/genius-scan/>
  - CombinePDF at <https://combinepdf.com/>

**GOOD LUCK!**

1. (6 points) Consider the power series solution for the following differential equation about the ordinary point  $x = 0$ .

$$(1 - x^2)y'' + y = 0$$

- (a) For what values of  $x$  can we guarantee that the series converge?
- (b) Find the recurrence relation for the coefficients of the series. Do not bother to calculate any coefficients.

2. (4 points) The power series solution for a particular differential equation has the following relationships between the coefficients.

$$\begin{cases} C_2 = \frac{-3C_0}{2} \\ C_{k+2} = \frac{-(k+3)C_k}{(k+2)(k+1)} \quad \text{for } k = 1, 2, 3, \dots \end{cases}$$

From this starting point, find two power series solutions of that differential equation about the ordinary point  $x = 0$ . Give the first three non-zero terms of each series (if they exist) and the general solution.

3. (5 points) Find the following Laplace and inverse Laplace transforms.

(a) Find the inverse Laplace transform  $\mathcal{L}^{-1}\{F(s)\}$  for

$$F(s) = \frac{3s + 4}{s^2 + 4}$$

(b) Find the Laplace transform  $\mathcal{L}\{f(t)\}$  for

$$f(t) = \begin{cases} t & \text{if } 0 \leq t < 6 \\ 0 & \text{if } t \geq 6 \end{cases}$$

4. (5 points) Use the Laplace transform to solve the following initial-value problem.

$$y'' - 6y' + 8y = e^{-3t}, \quad y(0) = 0, y'(0) = 0$$